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Effect of Inner Body Shapes on Natural Convection in Square Enclosures

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Abstract

This review focuses on the effects of inner body shapes on magnetohydrodynamic (MHD) natural convection heat transfer within square enclosures, summarizing advancements from 2015 to 2024 and identifying future research directions. The influence of different geometric shapes such as circular, square, triangular, and elliptical on heat transfer performance and flow behavior is critically examined. The review also addresses the roles of Rayleigh and Hartmann numbers in modulating convection characteristics in the presence of a magnetic field. Key trends and findings are highlighted, along with observed research gaps, to guide future studies aiming to enhance thermal management in MHD driven systems.

Keywords: Natural convection, Magnetic field, Square enclosures, Inner bodies, Hartmann number.

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1. Introduction

Natural convection is a critical heat transfer process driven by fluid motion from internal forces rather than external sources. This motion arises from buoyancy effects, where temperature-induced density gradients in the fluid initiate flow. Natural convection plays a vital role in numerous applications in engineering, including electrical component cooling, building insulation, solar energy systems, heat exchangers, material drying, and thermal storage. Unlike forced convection, it operates without input of external energy, which makes it inherently energy efficient. Understanding the mechanics of natural convection and evaluating thermal performance are therefore crucial for optimizing these systems. Due to the intricate interactions between the fluid and boundary layers on horizontal and vertical surfaces, this phenomenon presents a complex internal flow problem. A wealth of studies have explored natural convection's heat transfer processes across diverse fields, each aiming to enhance specific performance outcomes [1].

Nanofluids, comprising nanoparticles suspended in base fluids such as water or oil, exhibit enhanced thermal properties that surpass those of conventional fluids. These specialized fluids play a crucial role in advancing heat transfer across various high-demand industrial applications, including solar energy systems, nuclear reactors, and high-performance electronic devices. Their effectiveness in electronic cooling, particularly for critical components like microprocessors, enables sustained high-efficiency performance. The stable dispersion of nanoparticles within the fluid prevents common issues like sedimentation and conduit blockages, which are often associated with larger particles. Future research into nanofluids is anticipated to further enhance thermal efficiency, leading to the development of more compact and efficient heating and cooling systems across industries [2].

Kasaeian et al. [3] have highlighted an important focus in the field of heat transfer, particularly in exploring methods to enhance the performance of energy devices. Among the recent techniques investigated for this purpose are using nanofluids and porous media within heat exchangers. Nanofluids are defined as suspensions of solid nanoparticles in base fluids, such as water. This paper provides a thorough analysis of various studies examining the combined application of nanofluids and porous media within thermal systems, considering diverse geometrical structures and flow regimes.

Das et al. [4] examine internal natural convection heat transfer enclosures characterized by triangular, trapezoidal, parallelogrammatic, and curved walls, incorporating both fluids and porous media. The study also explores enclosures filled with nanofluids. Key parameters such as aspect ratio and the base angle of triangular and rhombic/parallelogrammatic enclosures significantly impact flow distribution. Furthermore, flow patterns are influenced by the number of undulations in the walls and the amplitude-to-wavelength ratio. The review highlights various strategies aimed at enhancing convection heat transfer performance, providing valuable insights into potential improvement techniques in this area.

Biswal and Basak [5] present a comprehensive review focusing on the investigation of entropy production in natural convection across a range of applications and geometries. The review elucidates the mathematical formulation of fundamental equations, calculation procedures, and methodologies for conducting thorough evaluations. A central challenge addressed is the trade-off between minimizing entropy generation and maximizing heat transfer rates for optimal configurations that enhance energy efficiency. The process of natural convection is crucial for converting renewable energy into usable forms, making the analysis of entropy production an essential consideration in the design of effective energy systems. Furthermore, the article offers insights and discussions aimed at guiding future research on improving energy efficiency within renewable energy systems.

Sadeghi et al. [6] provide a comprehensive review of recent literature on the natural convection of nanofluids across various container geometries, including triangular, trapezoidal, square, circular, and non-traditional forms. The study explores the relationship between thermophysical properties and the geometric configuration of the cases, employing numerical methods and microscopic models to support their findings.

Giwa et al. [7] investigate the control of Heat transmission via spontaneous convection and the flow characteristics of nanofluids in square cavities utilizing magnetic field sources. The study examines the influence of various parameters, For the magnetohydrodynamic (MHD) behaviors of natural convection, these include heat distribution techniques, thermal and concentration boundary conditions, governing parameters, magnetic field types, numerical schemes, thermophysical correlations, nanofluid types, slip conditions, Brownian motion, and thermophoresis.

Hemmat et al. [8] review of natural convective heat transfer in nanofluid-filled cavities influenced by Magnetic Fields (M.F). The paper provides an introduction to nanofluids and their applications, along with an evaluation of numerical studies on magnetic nanofluids. It classifies the literature based on common geometries and effective parameters relevant to nanofluids under free convection and magnetic field effects. The findings indicate that when the magnetic field strength increases, the rate of heat transmission falls.

As the field of natural convection continues to evolve, numerous researchers have contributed to the understanding of heat transfer mechanisms in various geometries and configurations. These studies have significantly advanced our knowledge, exploring factors such as the impact of nanofluids, geometric shapes, and magnetic fields on natural convection performance. The following section will highlight key findings and methodologies from previous studies, offering a comprehensive overview of the advancements in this area.

2. The most selected parameters in the previous publications

A substantial body of literature discusses natural convection in square enclosures, looking at a number of variables such as the Rayleigh number (Ra), which quantifies the ratio of buoyancy forces to viscous forces. Studies, including those conducted by Chen et al. [9], have demonstrated that increasing the Rayleigh number correlates with an enhanced pace at which heat is transferred by natural convection.

Flow patterns and the amount of heat transferred are significantly influenced by the Rayleigh and Hartmann numbers, which serve as critical variables in determining flow behavior; an increase in the Rayleigh number results in more complex flow structures and consequently enhances the heat transfer rate. Studies by Chen et al. [9] show that because a magnetic field inhibits fluid motion, raising the Hartmann number (Ha) lowers heat transfer rates within square cavities. [9, 10]. Numerical modeling and experimental analysis are essential for understanding these dynamics. Techniques For example, the Finite Volume Method (FVM) and the Finite Element Method (FEM) offer valuable predictive insights into temperature variations within the cavity under changing parameters. However, experimental validation is necessary to ensure the reliability of numerical results against actual data, thus enhancing the credibility of the findings [9, 10].

2.1. Conventional Square enclosure in the absence of the magnetic field

Joshi and Pattamatta [11] conducted an experimental investigation on buoyancy induced Using AL_2O_3/Water and MWCNT/Water nanofluids in a square cavity for convective heat transfer. Their findings revealed that the MWCNT/Water nanofluid exhibited higher Nusselt numbers compared to the AL_2O_3/nanofluid of water at certain volume fractions. The volume ratios used to create the MWCNT/Water nanofluid were 0.1%, 0.3%, and 0.5%.and the study analyzed its performance across a Rayleigh number range from 7 × 10^5 to 1 × 10^7.

Hidayathulla et al. [12] conducted Examining unstable heat and momentum transfer numerically in a Newtonian fluid that fills a square hole. Their analysis the governing equations are solved using the Harlow-Welch Marker and Cell (MAC) finite difference method. In their study, they varied the thermal Grashof number (Gr) while maintaining a Prandtl number (Pr) of 0.7 (for air) and a Reynolds number (Re) of 10, indicating laminar flow conditions.

Khatamifar et al. [13] investigated the heat transfer from transient conjugate natural convection in a differentially heated cavity numerically, considering the effects of finite partition thickness and thermal conductivity. The study explored a range of Rayleigh numbers (Ra) from 10^3 to 10^8, with dimensionless partition thicknesses between 0.05 and 0.2 and the partition's thermal conductivity ratio to the liquid between 0.1 and 1000. The results illustrate the transient development of natural convection flow within the partitioned cavity for various values of Ra (10^3, 10^4, 10^5, 10^6, 10^7 and10^8), k_r(0.1, 1, 10, 100, 500 and 1000) and T_P (0.05, 0.1 and 0.2), all maintaining a constant X_P = 0.5, H/L = 1, and Prandtl number (Pr) of 0.71.

Kumar et al. [14] investigated the beginning of the free thermal flow of a silver nanofluid in a closed square chamber with isothermal vertical walls at different temperatures and adiabatic horizontal walls filled with a saturated porous media. Using the Boussinesq approximation and the Darcy model, they used a two-temperature nonequilibrium model to construct the governing boundary layer equations. Setting Γ = 1, γ = 1, H = 10, Ra = 100, and R_d = 0.3 as default settings for the emerging flow parameters was done by the study.

2.2. Conventional Square Enclosure without Inner Body in the Presence of a Magnetic Field

Many researchers have examined the impact of magnetic fields on natural convection heat transfer within square enclosures. For instance, Mansour and Bakier [15] a computational investigation of the flow of nanofluids and the heat transfer related to natural convection in the presence of an angled magnetic field and varied thermal boundary conditions. They looked into a lot of different things, like the heat source's length (0.2 < B < 0.8), its location (0.3< D < 0.7), the solid volume fraction (0 < φ < 0.2), the inclination angle (0 < Φ < 90°), the Rayleigh number (10^3 < Ra < 10^5), the Prandtl number (Pr = 6.2), and the Hartmann number (0 < Ha < 100). The results indicated that the inclination angle significantly affects heat distribution; as the angle increases, convection is enhanced, and the presence of nanoparticles improves conduction, thereby increasing the heat transfer rate. The study concluded that the temperature distribution within the cavity is influenced solely by the volume fraction of nanoparticles, a finding supported by numerous prior studies.

Furthermore, the article presents a numerical investigation of natural convection under the influence of magnetic fields in a square container containing an ethylene glycol-copper nanofluid, employing COMSOL Multiphysics for simulation. The authors concluded that the impact of the Hartmann number on the y-directional velocity component are more pronounced than those of the Rayleigh number. With a Prandtl number set at Pr = 151, they inspected the impacts of the relevant parameters, specifically the Rayleigh and Hartmann numbers, on the flow dynamics and heat transfer performance within the enclosure. The study also highlighted that enhanced ethylene glycol nanofluids exhibited superior thermal conductivity compared to other nanoparticles, as noted by Ben Hamida and Charrada [16].

Sreedevi and Reddy [17] looked over heat transport by natural convection in a square cavity with a Tiwari-Das nanofluid model, considering magnetic field and thermal radiation. The system's nonlinear partial differential equations were solved using the finite difference method. Their study investigated the impact of a number of critical parameters on the flow and heat transfer properties of the TiO_2-EG nanofluid, such as the volume fraction ($0.01 \leq \emptyset \leq 0.09$), the magnetic field parameter ($1.0 \leq M \leq 3.0$), the Rayleigh number ($100 \leq Ra \leq 1000$), the radiation parameter ($0.1 \leq R \leq 0.9$), the Reynolds number ($0.1 \leq Re \leq 0.5$), the Rayleigh number ($1.0 \leq Pr \leq 7.2$), and the Prandtl number ($5.2 \leq Pr \leq 7.2$). Graphs show how these factors affect the behavior of nanofluids and how they affect the performance of heat transfer and flow.

Haritha et al. [18] analyzed free convection heat transfer in a porous medium-filled square cavity saturated with nanofluids containing various nanoparticles, under the influence of an applied magnetic field and viscous dissipation. Using Darcy's model, they applied the finite element method based on the weighted Galerkin residual scheme to solve the governing equations. Parameters included Rayleigh number (Ra = 50, 200, 800), magnetic field parameter (M = 0.5, 1, 1.5), and nanofluid volume fraction (ϕ = 0.02, 0.2, 0.6). The study found that heat transfer and flux density were highest with Cu nanoparticles and lowest with AL_2O_3, with Figure 1 illustrating changes in heat transfer for varying Eckert numbers and volume fractions of nanoscale copper (ϕ= 0.02, 0.2, 0.6).

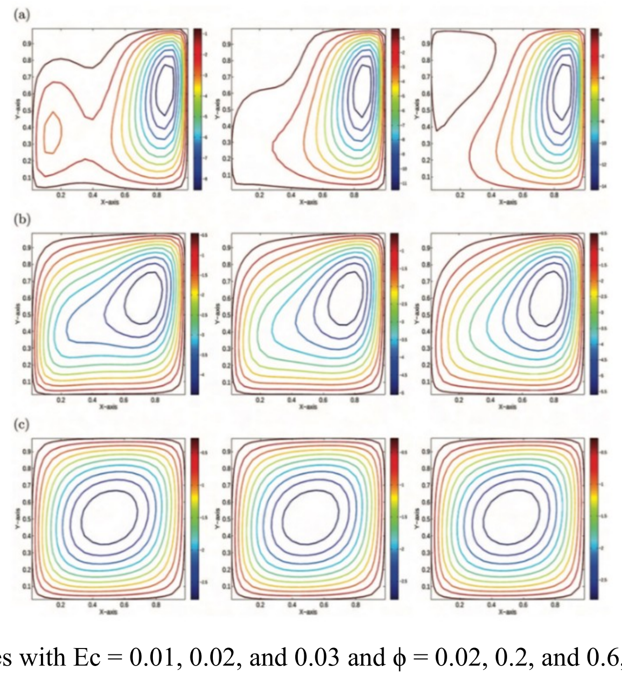


Figure 1. streamlines with Ec = 0.01, 0.02, and 0.03 and ϕ = 0.02, 0.2, and 0.6, respectively [18].

Mehryan et al. [19] showed how a periodic magnetic field affected the Fe_3O_4 nanofluid's natural convection and entropy production in a square cavity. To solve the governing partial differential equations, they used the Galerkin finite element method, concentrating on dimensionless parameters like the nanoparticle volume fraction (φ = 0-0.08), period number (λ = 0.1 - 1.0), Rayleigh number (Ra = 10^3-10^6), and Hartmann number (Ha = 0-50), which indicates the amplitude of the magnetic field. The results showed that total entropy production (St) under the periodic magnetic field was higher than that under a uniform magnetic field, regardless of Ha and λ values.

Mansour et al. [20] treated entropy generation, MHD convective flow, and heat transfer in a square porous cavity. Using finite difference methodology, they assessed parameters such as the magnetic field (0 ≤ Ha ≤ 100), heat source (−4.0 ≤ Q ≤ 4.0), volume fraction of nanoparticles (0.03 ≤ ∅ ≤ 0.1), and permeability ($10^{-6} \leq$ Da $\leq 10^{-2}$). Their findings showed that adding copper nanoparticles achieved the highest heat transfer rates among the materials tested.

Al Kalbani et al. [21] studied the effects of a directed magnetic field on the heat transfer and fluid flow properties of natural convection in an inclined square container filled with different nanofluids containing different forms of nanoparticles. The governing non-dimensional partial differential equations were solved using the weighted Galerkin residual finite element technique. Their study considered different values for the following: inclination angle (0° ≤ δ ≤ 90°), Hartmann number (0 ≤ Ha ≤ 60), magnetic field orientation (0° ≤ y ≤ 90°), Rayleigh number ($10^3 \leq$ Ra $\leq 10^6$), and nanoparticle volume percent (0 ≤ ∅≤ 0.05).

Mahapatra and Parveen [22] analyzed spontaneous convection flow in a copper-water nanofluid-filled container with a sinusoidal top wall that was differentially heated and subjected to a continuous vertical magnetic field. A constant Prandtl number of Pr = 6.2 was maintained throughout their investigation as they investigated a number of parameters, such as Rayleigh number ($10^2 \leq$ Ra $\leq 10^6$), sinusoidal wall amplitude (0 ≤ a ≤ 0.4), nanoparticle volume fraction (0.0 ≤ ϕ ≤ 0.2), and Hartmann number (0 ≤ Ha ≤ 50), using the Bi-CGStab method for numerical simulations. Increasing the Rayleigh number from 10^2 to 10^4 improved flow strength, heat transfer rate, and entropy formation, according to their findings, both with and without magnetic fields.

Uddin et al. [23] assessed natural convection heat transfer using a heterogeneous dynamical model in a square jar filled with a copper oxide nanofluid, exposed to a uniform magnetic field, and having a wavy top wall. They discussed the effects of several control parameters on the flow and thermal fields, such as Hartmann number, magnetic field inclination angle, gravitational inclination angle, solid volume fraction, nanoparticle diameter, and dimensionless time, by applying the Galerkin finite element method to the governing equations. Their study's main goal was to assess the improvement in heat transmission under various parameter combinations for real-world uses.

Rajarathinam and Chamkha [24] investigated the effects of partial convection in a water-based nanofluid within a square cavity subjected to a magnetic field. They addressed the governing unstable non-dimensional partial differential equations using the SIMPLE algorithm in conjunction with the finite volume method. Their results, presented in Figure 2, encompass various relevant parameters, including the Hartmann number ($0 \leq Ha \leq 100$), Rayleigh number ($10^2 \leq Ra \leq 10^6$), solid volume fraction ($0.0 \leq \phi \leq 0.04$), and three distinct opening configurations. The study reveals that the vertical velocity component at the cavity's center achieves its maximum when both buoyancy forces and velocity boundary conditions are absent, influencing the flow behavior across all positions.

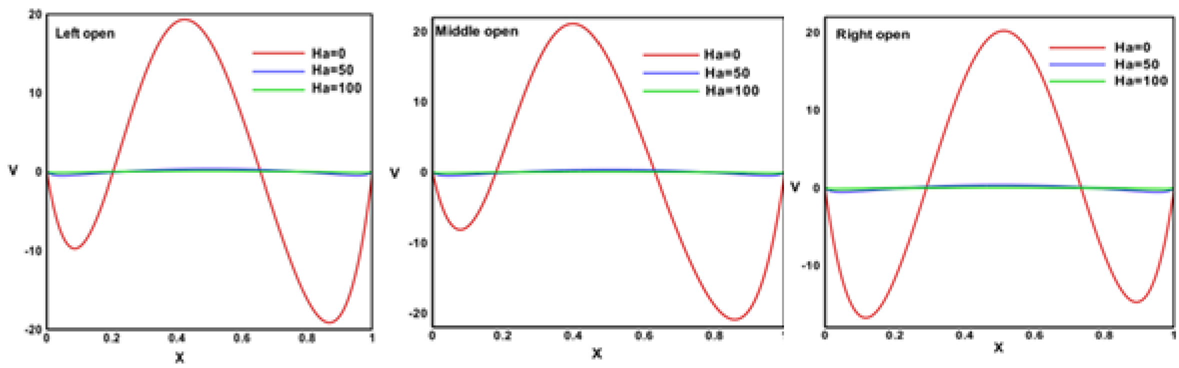


Figure 2. Vertical velocity profiles for various Hartmann numbers and opening positions at Y= 0.5, with $Ra = 10^4$ and $\phi = 0.02$. [24]

Devi et al. [25] explored the influence of magnetic wire on the viscous Casson flow in a container with opposing temperature gradients and cadaveric upper and lower walls, utilizing the MAC technique for numerical analysis. The work concentrated on controlling the following fluid flow parameters: Casson fluid parameter $\beta = 0 - \infty$, Hartmann number (0 - 8), Rayleigh number ($10^3 - 10^5$), and a constant Prandtl number $Pr = 6.8$. According to their research, buoyant force causes the temperature gradient to rise.

Nishad et al. [26] analyzed heat transfer and flow in a copper-water nanofluid-filled undulating container while subjected to a magnetic field using a parallel grid-free method. Using the element-free Galerkin method (EFGM), the equations governing the transport phenomena were numerically solved. Their findings were obtained for a number of parameters, such as the magnetic field inclination angle ($0° \leq g \leq 80°$), nanoparticle volume fraction ($0 \leq w \leq 0.5$), Rayleigh number ($10^3 \leq Ra \leq 10^5$), and

Hartmann number (0 ≤ Ha ≤ 60). The study found that a greater Hartmann number Lorentz force suppresses fluid motion and lowers the cavity's heat transfer rate, which in turn reduces stream functions.

2.3. Previous studies with inner Bodies

2.3.1 Square inner Body

Munshi et al. [27] performed a numerical analysis of a square's natural convection container characterized by an irregularly unheated bottom wall and a hot mass in the shape of a square. They explored the influence of various Prandtl numbers (0.71, 1.0, and 1.5) on Heat transmission and fluid flow within the enclosure. Their parametric analysis revealed that as the Prandtl number increases, free convection is suppressed, leading to a predominance of heat transmission via conduction. The results indicated that the heat transfer processes, temperature distribution, and flow characteristics inside the cavity are significantly influenced by the Rayleigh number as well as the magnetic field's strength.

Boulahia et al. [28] performed a numerical investigation into the natural thermal transfer of a nanofluid (Cu-water) within a square enclosure containing a cold rectangular obstacle. They employed the finite difference method to solve the transport equations using the Alternating Direction Implicit (ADI) approach. Their study identified several dimensionless groups, including the height of the obstacle (0.125 ≤ H ≤ 0.5), the Rayleigh number ($10^3 \leq$ Ra $\leq 10^6$), and the volume fraction of nanoparticles (0 ≤ φ ≤ 0.2), while maintaining the obstacle's width at 0.25L and the Prandtl number for pure water at Pr = 6.2. The findings demonstrated that heat transfer was enhanced by increasing both the Rayleigh number and the volume fraction of nanoparticles.

2.3.2 Elliptical inner body

Munshi et al. [29] examined how fluid flow and heat transfer were affected by magnetohydrodynamic (MHD) natural convection in a square cavity that held an electrically conducting fluid with an elliptical adiabatic mass. Using numerical techniques, the study produced predictions over a wide range of Hartmann numbers (Ha) and Rayleigh numbers (Ra) while keeping the Prandtl number constant at 0.733. The governing equations pertaining to the parameters of heat production were solved using the finite element method. The findings indicated that an increase in the Rayleigh number enhances the buoyant force, necessitating a stronger magnetic field to reduce natural convection effects, particularly at lower Rayleigh numbers.

Adegun et al. [30] conducted a numerical investigation of normal convective heat transfer and fluid flow within a concentric square ring featuring an internal inclined elliptical cylinder subjected to isothermal heating and cooling, as illustrated in Figure 3. The researchers employed the Galerkin finite element method to solve the governing elliptic conservation equations. The parameters evaluated included Rayleigh numbers ranging from 10^3 to 10^3 and elliptical orientation angles varying from 0° to 90°. Additionally, an aspect ratio was considered, with values ranging from 1 to 3. This study provides insights into the effects of geometric configuration and thermal conditions on convective heat transfer characteristics.

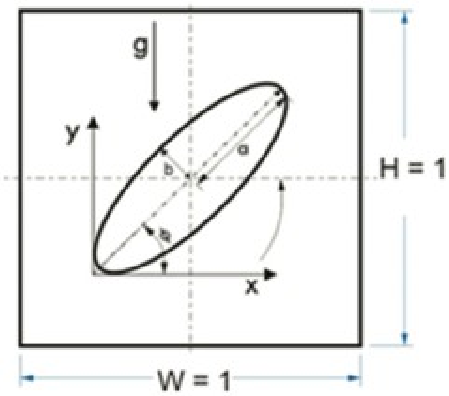


Figure 3. Physical coordinate geometry. [30]

Cho, Ha et al. [31] investigated two-dimensional natural convection numerically in a square container with a vertical arrangement of two elliptical cylinders. They used the Immersed Boundary Method (IBM), which combines the FVM and Immersed Boundary Method, to precisely depict the cylinders' virtual wall limits. There was a range of 0.25 to 4.00 for the elliptical cylinders' aspect ratio (AR) and 10^4 to 10^6 for their Rayleigh number (Ra). The study's main objective was to examine how the elliptical cylinders' varying aspect ratios affected the flow and heat fields.

Zhang et al. [32] focused on the numerical study of steady natural convection in a cold outer square enclosure containing a hot inner elliptical cylinder. They utilized the multiscale element-free Galerkin (VMEFG) method to conduct their numerical investigations. The study maintained a constant Prandtl number (Pr = 0.71) and an eccentricity of $\varepsilon = 0.9$ while examining different values of the dimensionless major axis (a = 0.2, 0.3, 0.4), the inclined angle of the outer square enclosure (γ = 0°, 15°, 30°, 45°), and the Rayleigh number (Ra = 10^3, 10^4, 5×10^4, 10^5, 10^6). Their study's findings are shown in Figure 4, which highlights the intricate relationship between the system's thermal dynamics and geometric design.

Park et al. [33] considered two-dimensional natural convection numerically in a square enclosure with a vertical array of heated circular and elliptical cylinders, with Rayleigh numbers between $10^4 \leq \mathrm{Ra} \leq 10^6$. Their study kept the Prandtl number constant at 0.7. The cylinder walls' borders were precisely captured by them using an immersed boundary approach. Their primary objective was to determine the impacts of the elliptical cylinder's inclination angle in this design statistically (Figure 5).

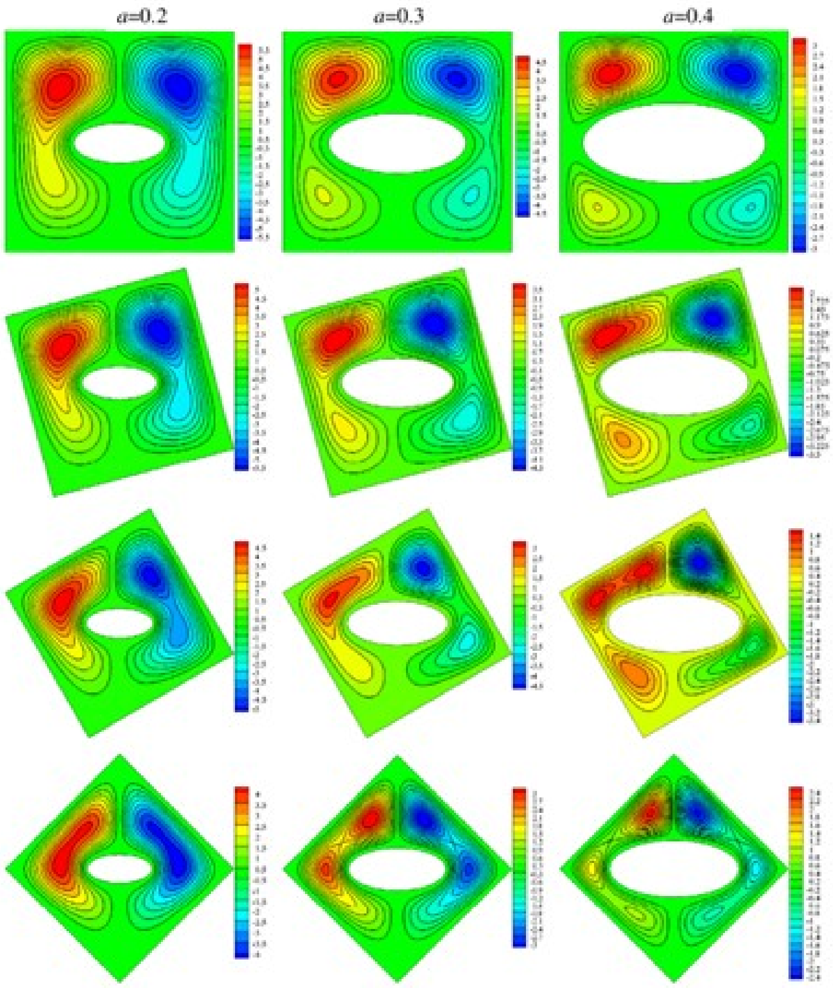


Figure 4. Streamlines for various a and γ values when Ra = 5×10^4 [32]

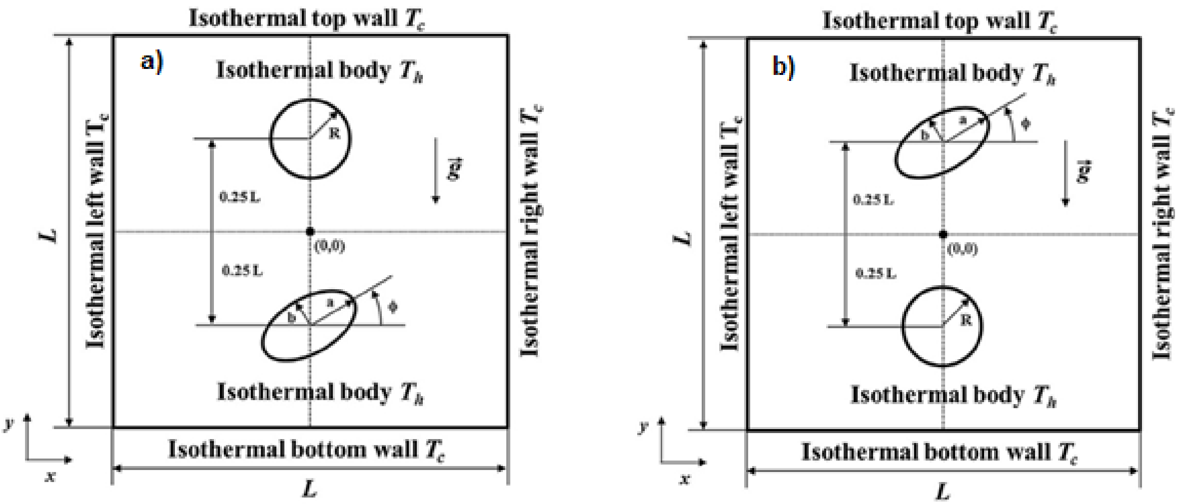


Figure 5. Physical model boundary conditions, coordinate system, and computational domain: (a) lower elliptical cylinder; (b) upper elliptical cylinder.[33]

Ibrahim et al. [34] studied natural convection inside cavities to introduce nanoparticles into the core fluid to improve heat transmission in various package forms. For their numerical computations, they used the COMSOL software, which is based on the Galerkin finite element method. The following parameters were used: the radius of the inner circle (R = 0.15), the radii of the inner elliptical cylinder (R_x = 0.2 and R_y = 0.15), the inclination angles (-45°,-30°, 0°, 30°, 45°), the solid volume percent ($\emptyset$ = 0.05), and the Rayleigh number Ra (varying from 10^3 to 10^6).

2.3.3 Inner circular body and cylinder

The magneto-hydrodynamic (MHD) heat transfer problem in a square open cavity containing a heated circular cylinder at the center has been investigated by Hossain et al. [35]. This study's objective is to explain how MHD affects flow and thermal fields when a heated circular cylinder is present, as visualized through graphical representations. They employed a numerical technique based on the weighted Galerkin residual method for finite element formulation. Heat transfer and fluid movement within the cavity depend on the Rayleigh number (Ra), Hartmann number (Ha), and heat flux (q). In this inquiry, the Prandtl number is fixed at Pr = 0.72 and the heat flow q = 100. The ranges for Ra and Ha are 10^3 to 10^4 and 0 to 400, respectively.

Shruti et al. [36] evaluated the combined effects of variations in Darcy and Rayleigh numbers on natural convection around two vertically arranged heated porous cylinders of different diameters in a square container. They employed the D_2Q_9 model to perform numerical simulations using a Boltzmann network. The following variations in flow and heat transmission were analyzed in relation to the Darcy and Rayleigh numbers and the cylinder volume: Porosity (ε) = 0.629, 0.977, and 0.993, that correspond to the related Darcy numbers; Rayleigh number (Ra) = 10^4, 10^5, and 10^6; and Darcy number (Da) = 10^{-6}, 10^{-4}, and 10^{-2}.

2.3.4 Inner cross shape

Ahmed and Aly [37] performed numerical simulations of natural convection induced by a hot cross-shaped body filled with stationary and moving solid particles. They utilized the improved incompressible particle hydrodynamics (ISPH) method to simulate the buoyancy-driven flow inside the enclosure. The key parameters in their investigation included the cross length L_{cross} (ranging from 0 to 0.8), solid particle depth L_{solid} (ranging from 0.02 to 1.8), time variations τ (ranging from 0.002 to 0.3), and the Rayleigh number Ra (ranging from 10^3 to 10^5). Their results indicated that the configuration with cold and moving solid particles resulted in the highest heat transfer rate, while the scenario with fixed and cold particles produced the lowest rate.

2.3.5. Square enclosure with two wavy sided walls

Ali et al. [38] conducted a numerical investigation of the flow and heat transfer components of buoyancy-driven convection in a hybrid cavity filled with nanofluids (Figure 6). The non-dimensional governing equations were solved using the finite element method. The simulation changed a variety of factors, such as the heater length (0.2 L to 0.6 L), the solid heat generation cylinder's radius (0.05 L to 0.2 L), the hybrid volume fraction (0% to 5%), the Rayleigh number (10^4 to 10^7), and the Hartmann number (0 to 50). With a constant Prandtl number of 6.2 and a thermal conductivity ratio of 5, the study found that increasing mesh configurations had minimal impact on the average Nusselt number, resulting in a satisfactory solution with 17,164 nodes and 33,554 elements (Figure 7).

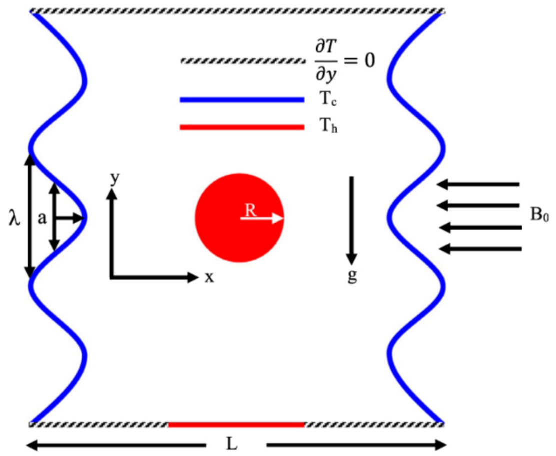


Figure 6. The current study's physical model. [38]

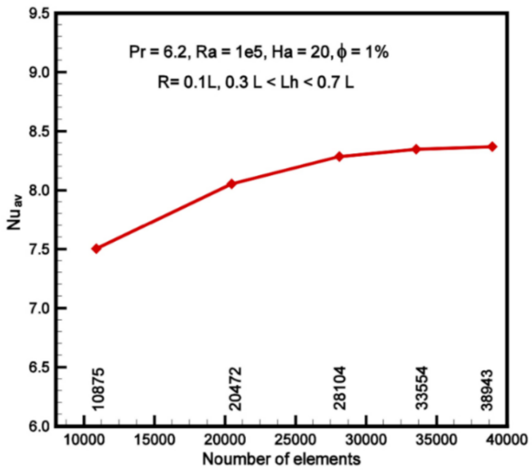
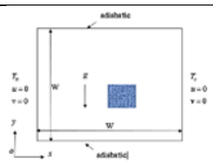
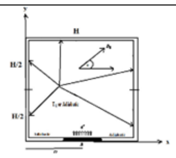
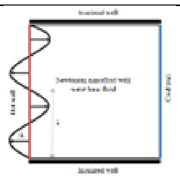
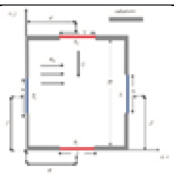
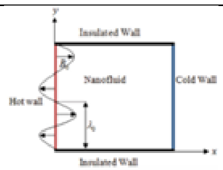
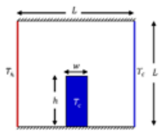


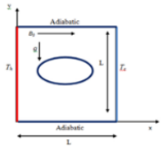
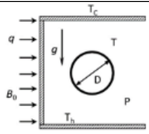
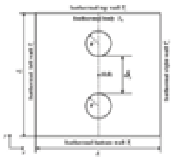
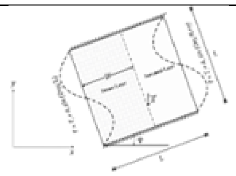
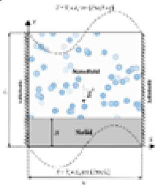
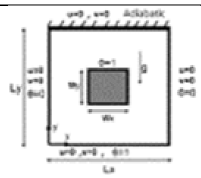
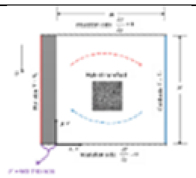
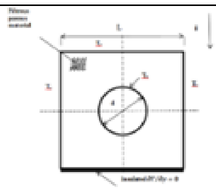
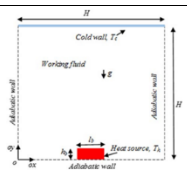
Figure 7. Grid refinement test. [38]

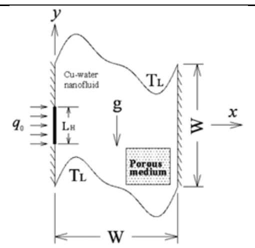
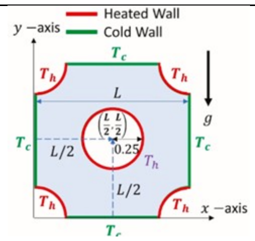
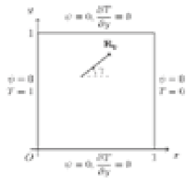
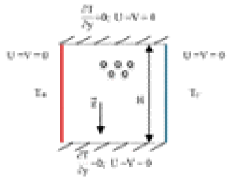
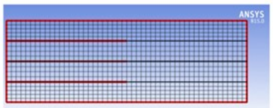
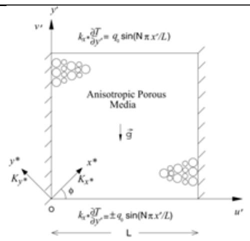
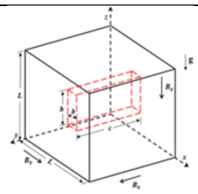
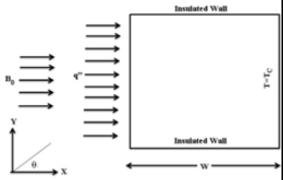
2.4. Triangular inside a square with MHD

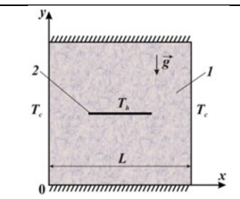
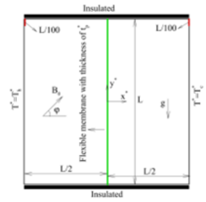
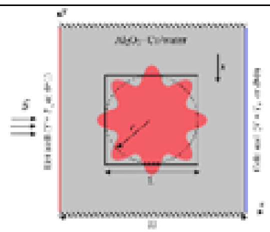
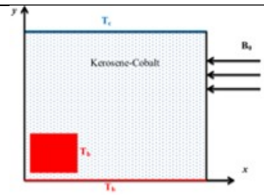
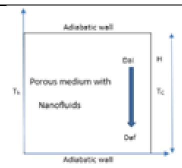
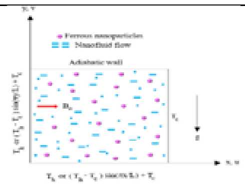
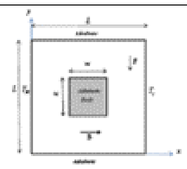
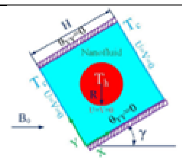
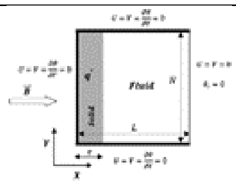
Vijaybabu and Dhinakaran [39] explored, using the Boltzmann method, the natural convection heat transfer between a cold square cavity and a hot, permeable triangular cylinder affected by a magnetic field.. The study studied the effects of key parameters, specifically Rayleigh numbers ranging from 10^4 to 10^6 and Darcy numbers from 10^{-6} to 10^{-2}. For Hartmann numbers Ha= 0, 25, and 50, the results are shown using streamlines, velocity profiles, isotherms, and local, surface, and mean Nusselt numbers. The results indicate that the high viscous resistance within the porous triangular cylinder at Da = 10^{-6} effectively blocks fluid flow through it, regardless of Rayleigh or Hartmann values.

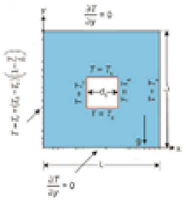
Table 1. A summary of the Effect of Different Internal and external bodies in Square Containers on Natural Convection

Reference	Major topic	Enclosure shape	Results
[14]	Natural convection in silver nanofluid		- Increasing H and y strengthens solid isotherms - Average Nusselt number increases.
[15]	MHD Natural convection		- Average Nusselt number increases with solid volume fraction.
[19]	MHD entropy generation in a ferrofluid		- Periodic magnetic field leads to higher total entropy than uniform field - Effect is consistent across Ha and λ values
[20]	MHD convection		- Fluid speed decreases significantly with increasing Hartmann number
[26]	MHD convection with nanofluid		- Heat transfer rate increases with magnetic field inclination angle (up to critical angle)
[28]	Natural convection with cold obstacle		- Heat transfer improves with increasing Rayleigh number, nanoparticle volume fraction, and cold mass height

[29]	MHD convection with elliptic shape		- Buoyant force increases with Rayleigh number
[35]	MHD free convection		- Heat flow decreases with increasing Hartmann number
[40]	Double inner cylinders		- Flow transitions from stable to unstable at Ra = 10^6.
[41]	Nanofluid convection in porous media		- Stable natural convection behavior in inclined porous media
[42]	Conjugate natural convection		- Phase deviation significantly alters flow and temperature distribution
[43]	Hot obstacle convection		Nusselt number decreases on cold walls - Shape coefficient increases as surface ratio decreases
[44]	Hybrid nanofluid flow		- Local Nusselt number sharply drops from bottom to top of cavity
[45]	Heated cylinder convection		- Average Nusselt number increases with thermal conductivity ratio at specific Grashof numbers
[46]	Nanofluid with thermal source		- Nanoparticles <6% enhance heat transfer at high Ra and inclination angles <30°

[47]	Wavy surfaces		- Nu_m and Be increase while S_t decreases with increasing φ and aa_w.
[48]	Round barriers & curved corners		- Central temperature increases by 400% as barrier radius grows from 0.1 to 0.3
[49]	MHD convection under slope		- Stream function and vorticity decrease with Ha - Increase with Ra
[50]	Alumina nanofluid convection		- Heat transfer improves with higher solid volume fraction and Rayleigh number
[51]	Finned nanofluid cavity		- Heat transfer enhanced by increasing Rayleigh number, fin number, location, and length
[52]	Sinusoidal heating		- Flow strength decreases when N > 1
[53]	3D convection		- Vertical plates transfer heat more efficiently than horizontal plates across Ac and Ha
[54]	Entropy and inclination effects		- Nusselt number and entropy more sensitive to tilt angle (θ) than Hartmann number

[55]	Heated plate in porous cavity		- Higher radiation parameter, plate length, and Darcy number enhance heat transfer
[56]	MHD convection with flexible membrane		- Hartmann number changes affect heat transfer and membrane shape
[57]	Hybrid nanofluid & wavy cylinder		- Convective flow increases with Ra and conductivity - Decreases with Hartmann number and undulation effects
[58]	Ferrofluid in porous cavity		- Nusselt number increases with Darcy and Rayleigh numbers - Decreases with Hartmann number
[59]	Nanofluid in porous cavity		- Low permeability reduces flow and heat transfer - High permeability improves nanofluid mobility
[60]	Uneven wall heating		- Flow rate declines with higher nanoparticle concentration and Hartmann number
[61]	MHD with adiabatic flow		- Magnetic field significantly alters flow at high Prandtl and Hartmann numbers
[62]	Inclined cavity with circular baffle		- Heat transfer rate increases on right wall and decreases on left with increased inclination angle
[63]	Uneven heat source with solid wall		- Fluid flow strength rises with Ra_I regardless of k_r

[64]	Heated cylinder		- Right-wall local Nusselt number increases with Ra - Stronger eddies observed

2.5. Inner body studies without MHD

Cho et al. [40] conducted a numerical investigation into two-dimensional natural convection within a square container, examining various configurations of two internal cylinders at Rayleigh numbers ranging from $10^3 \leq Ra \leq 10^6$.. Their simulations, based on the immersed boundary method, provided precise solutions. Results indicated that, at lower Rayleigh numbers, the solutions remained stable regardless of variations in δ. However, when the Rayleigh number increased to Ra = 10^6, the solutions became unstable for configurations with $\delta_h = 0.1$ L., and δ_d=0.2L. Additionally, the average Nusselt number $< \overline{(Nu_c)} >$ increased by approximately 29.2%, 12.8%, and 30.8% for the cases of δ_v, δ_h, and δ_d, respectively. Similarly, the average values of $< \overline{(Nu_{En})} >$ rose by around 27.7%, 12.4%, and 31.0% in these configurations.

Alsabery et al. [41] used a finite difference approach to study natural convection heat transport in an inclined square cavity with a porous layer and a nanofluid. The study concentrated on the effects of the following important parameters: cavity inclination angle (0°≤ φ ≤ 90°), phase deviation (0 ≤ γ ≤ π), amplitude ratio (0 ≤ ε ≤ 1), porous layer thickness (0.1 ≤ S ≤ 0.9), the Darcy number (10^{-5} ≤ Da ≤ 10^{-3}), and nanoparticle volume fraction (0 ≤ $\emptyset$ ≤ 0.2). Rayleigh number (Ra_{bf} = $10^4, 10^5$) and Prandtl number (Pr_{bf}= 6.2). Average Nusselt number values were computed for various φ and S values, revealing significant flow structure enhancement at lower inclination angles. The flow was observed as a cell in the nanofluid layer that rotates clockwise.

In another study, Alsabery [42] investigated conjugate natural convection in a square cavity including sinusoidal temperature fluctuations along the horizontal walls, focusing on a nanofluid devoid of an interior body. The study employed the finite difference method to investigate the effects of various parameters, including Rayleigh number (10^5 ≤ Ra ≤ 10^8), nanoparticle volume fractions (0 ≤ ϕ ≤ 0.2), phase deviations (0 ≤ ϒ ≤ π), amplitude ratio (0 ≤ ε ≤ 1), wall-to-nanofluid thermal conductivity ratio (0.44 ≤ k_r≤ 23.8), and wall thickness-to-height ratio (0 ≤ S ≤ 0.7).

Alturaihi et al. [44] explored natural convection within a heated cylinder located in a square cavity filled with a porous medium. Key parameters, including the Prandtl number (Pr = 0.7) and Darcy number (Da = 0.01), were selected for analysis. Five levels of porosity (ε = 0.4, 0.5, 0.55, 0.6 and 0.65) were considered, in addition to different values of the thermal conductivity ratio (kr) and Grashof number (Gr). Their findings revealed that increased porosity significantly enhances convective heat transfer within the cavity by strengthening fluid flow patterns. In a related study, Ghalambaz et al. [45] numerically investigated Spontaneous natural convection in a square cavity containing a hybrid nanofluid of Ag, MgO, and water. They treated Rayleigh numbers between 10^3 and 10^5 and a nanoparticle volume percent between 0 and 0.02 with a fixed Prandtl number of Pr = 6.2 by solving the governing equations using the finite element method. According to their findings, hybrid nanofluids do not always improve the cavity's natural convection heat transmission.

El Mehdi et al. [46] studied the lattice Boltzmann flow behavior of a Cu/water nanofluid in a square cavity. Their simulations covered various Rayleigh numbers (from 10^5 to 0.5×10^7), cavity inclination angles (0° to 90°), and nanoparticle volume percentages (0 – 8%). The study assessed the effects of these parameters on fluid rheology and isothermal distributions within the cavity. Findings revealed that adding 8% nanoparticles effectively prevented flow separation.

Cho et al. [47] presented the natural convection of nanofluids in a porous cavity with wavy top and bottom surfaces and a vertical wall that is partially heated. They conducted numerical simulations to examine the effects of several parameters, including the irreversibility distribution (χ) divided by the Bejan number (Be), the length of the heated wall surface (L_H^*), the surface wavy amplitude (a_w), the Rayleigh number (Ra), the Darcy number (Da), the porosity (ε), and the nanoparticle volume fraction (φ). The average Nusselt number (Nu_m), energy flow vector distribution, and total entropy generation (S_t) were among the main topics of their investigation. Using the Tri-Diagonal Matrix Algorithm (TDMA), they were able to solve the governing systems.

2.6. Conventional Enclosure without inner body

Tezer-Sezgin et al. [49] surveyed natural convection in a square container containing hydrated aluminum oxide (Al_2O_3) when an external oblique magnetic field was present. This study employed two numerical techniques, specifically DRBEM, or Dual Reciprocity Boundary Element Method, and FEM, or Finite Element Method, utilizing different meshing types. The research focused on the effects of key flow parameters, including Rayleigh (Ra) and Hartmann (Ha) numbers, inclination angle (y), and solid volume fraction (ϕ) are among them. Rayleigh and Hartmann numbers were as high as 10^7 and 300, respectively, for numerical simulations with inclination angles of y = 0, π/4,π/3, and π/2 with a solid volume fraction range of 0≤ ϕ ≤ 0.20. Notably, as illustrated in Figure 8, the DRBEM achieved mesh independence with 200 fixed boundary elements, while the FEM used 1152 quadratic triangular elements to ensure similar independence.

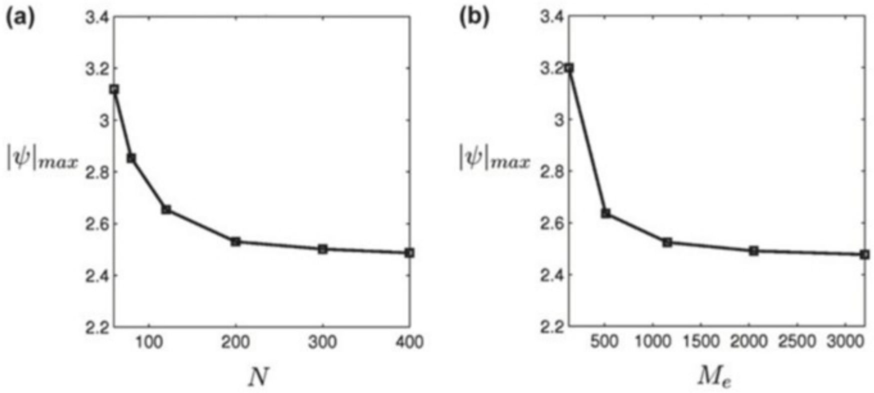


Figure 8. For Ha = 60, Ra = 10^5, and ϕ = 0.03 when γ = 0, the grid dependency is (a) DRBEM, (b) FEM.. [49]

Bouamoud and Houat [50] treated natural convection flow in two dimensions in a square hole with vertical walls that varied somewhat. Using a coupled population technique and the thermal grid Boltzmann method, they performed numerical simulations on a homogeneous nanofluid with alumina nanoparticles and pure water (Pr = 6.2) at solid volume fractions of φ = 0.02, 0.04, 0.06, and 0.08 in the laminar domain. According to their findings, the solid volume percentage of the nanoparticles has a major impact on the improvement of the heat transfer rate inside the cavity.

Chandra et al. [52] examined how different levels of polarization affected natural convection using the SIMPLER and Brinkman-extended-Darcy models, as well as the Alternating Direction Implicit (ADI) approach in certain situations. Eight non-dimensional parameters the Prandtl number (Pr), the Darcy number (Da), the Rayleigh number (Ra), the permeability ratio (K^*), the direction angle (ϕ), the thermal conductivity ratio (K^*), the periodicity parameter (N), and the porosity (ε) governed the velocity field and heat transfer rate, which they studied in relation to temperature.

Purusothaman et al. [53] investigated the finite volume method numerically for fluid flow and heat transfer in a cubic cavity caused by three-dimensional natural convection. Among the many parameters they calculated were the Hartmann number (0 ≤ H ≤ 300), the Prandtl number (0.025 ≤ Pr ≤ 25), and the plate's various aspect ratios (AC = 0.5 and 1.0). while keeping the Rayleigh number fixed at 10^7. They concluded that both the heat transfer rate and the flow characteristics within the cavity are significantly influenced by the strength of the magnetic field for Prandtl numbers greater than or equal to 0.71.

In another study, Mamourian et al. [54] investigated natural convection heat transfer and entropy generation in Al_2O_3 aqueous nanofluids within a square cavity subjected to an inclination angle and a constant axial magnetic field. They employed the governing equations are numerically solved using the finite volume approach. and utilized response surface methodology (RSM) for effective parameter analysis. The impacts of inclination angles (0°, 30°, and 90°), Hartmann numbers (0, 10, 30, and 50), and Rayleigh numbers (10^3, 10^5, and 10^6) were investigated. At θ = 0.05, they also investigated the effects of the inclination angle, Hartmann number, and Rayleigh number

Sivaraj and Sheremet [55] conducted A numerical simulation of thermal radiation and natural convection in a square porous cavity with a thin, isothermal heated plate sitting either vertically or horizontally in the middle. They solved the governing equations using an evenly layered grid structure and the finite volume approach. Their results showed that the radiation parameter, plate length, and Darcy number all considerably improve overall heat transmission in the cavity. A Rayleigh number of Ra = 10^7 was used to examine the effects of the radiation parameters ($0 \leq R_d \leq 2$), plate length (0.25 ≤ D ≤ 0.75), and Darcy number (10^{-5} ≤ Da ≤ 10^{-2})

Mehryan et al. [56] numerically investigated unstable natural convection in a square cavity separated by an elastic membrane that is impermeable. They modeled the fluid-membrane interaction using the arbitrary Lagrangian-Eulerian (ALE) technique in conjunction with the finite element method. Their study included a parametric analysis of key factors, including the Rayleigh number (10^5 - 10^8), Hartmann number (0 - 200), and magnetic field direction (0 - 180°). Their results indicated that the rotation of the fluid flow was enhanced with increasing magnetic field strength and directional adjustments.

2.7. inner body studies with MHD

Javed et al. [58] presented numerical results for free convection within a square container with a ferrofluid-saturated porous media inside, subjected to a constant magnetic field applied along the x-axis. They carried out numerical simulations using the finite element approach across a variety of flow parameters, such as the Rayleigh number, Hartmann number, Darcy number, and Prandtl number. Using constant values of Pr = 6.2, Ra = 10^6, Ha = 30, and nanoparticle volume fraction ϕ = 0.15, their study specifically assessed the effects of the Darcy number on heat transmission and flow architectures at different blockage locations. They observed that as the Darcy parameter increases, the strength of clockwise rotation intensifies, as indicated by the maximum current function magnitude (ψ). Specifically, ψ values reached 0.09, 1.06, and 1.89 for Da = 10^{-5} ,10^{-4}. and 10^{-3} respectively, in the case of left boundary condition (LBC), while in the right boundary condition (RBC), they were 0.007, 0.08 and 1.9 for Da = 10^{-5},10^{-4} and 10^{-3}, respectively.

Sivaraj and Sheremet [65] considered natural convection within an inclined porous cavity containing a centrally placed, heat-conducting solid body, under the influence of a magnetic field oriented from various directions. They used a finite volume method on a uniformly meshed grid to solve the coupled partial differential equations governing fluid flow and heat transfer. The study focused on the effects of Hartmann number ($0 \leq Ha \leq 50$), cavity inclination angle ($-45° \leq \xi \leq 90°$), and magnetic field inclination angle ($0° \leq \gamma \leq 180°$) on flow properties, isotherms, and the average Nusselt number. Results showed that the average Nusselt number peaks at a cavity inclination of $\xi = 30^{\circ}$ without a magnetic field, while in the presence of a magnetic field (Ha = 50), it reaches its maximum at $\xi = 45^{\circ}$.

2.8. Previous studies without inner body and Without MHD

Cherifa et al. [59] treated the Galerkin finite element method was used to solve the dimensionless equations and the Buongiorno model was used to study laminar natural convection in a porous square cavity filled with nanofluids. They analyzed various parameters with a cavity height of H = 2m, a Prandtl number of 5.82, Rayleigh number of 10^5, Darcy number of 10^{-2}, Lewis number of 1, and Brownian motion, thermophoresis, and buoyant force ratios all set to 0.1 ($N_r = N_b = N_t = 0.1$). Further investigations were performed across ranges of Rayleigh ($10^4 \leq Ra \leq 10^6$), Darcy ($10^{-5} \leq Da \leq 10^{-2}$), and an initial Darcy number that decreased to a final Darcy number of 10^{-5}.

Acharya et al. [60] explored the hydrothermal behavior of radiative aqueous nanofluid Fe_3O_4 within a square chamber. They employed the Galerkin finite element technique after transforming the main dimensional equations into dimensionless form using similarity variables. Their simulations checked the impacts of the following fixed values: Ra=10^4, Ha = 5, N = 0.5, $\phi = 0.02$, Pr = 6.2, thermal radiation ($0.5 \leq N \leq 1.5$), Rayleigh number ($10^3 \leq Ra \leq 10^5$), and Hartmann number ($5 \leq Ha \leq 25$).

Scott et al. [66] studied the impact of various volume concentrations of AL_2O_3-MWCNT (10:90) water-based hybrid nanofluids on heat transfer performance within a square cavity. Their findings indicated that hybrid nanofluids considerably outperformed single-nanoparticle nanofluids, with a maximum enhancement of 43.78% in heat transfer efficiency (h_{av}) at a concentration of 0.10 vol% at 50°C compared to deionized (DI) water. As Rayleigh numbers increased, the average Nusselt number (Nu_{av}) increased as well. for AL_2O_3-MWCNT hybrid samples across different concentrations and base fluids, highlighting hybrid nanofluids' potential as effective heat transfer fluids.

3. Conclusion

This review presents a comprehensive overview of recent studies on natural convection heat transfer (NCHT) within square enclosures, both with and without the presence of internal bodies, and examines the influence of magnetic fields on fluid flow and thermal performance. Covering research conducted between 2015 and 2024, the review highlights key advancements as well as gaps in current knowledge, emphasizing areas where further investigation could significantly advance the field.

The analysis identifies several promising directions for future research:

- While circular internal bodies have been extensively examined, elliptical geometries remain underexplored. Further studies in this area may uncover distinctive thermal behaviors arising from their unique flow patterns.
- Most existing literature focuses on single-body configurations, or occasionally, systems with two or four bodies. Extending this research to include multi-body arrangements could improve understanding of complex thermal interactions and contribute to optimized enclosure designs for enhanced heat transfer.
- Future investigations should consider how varying magnetic field strengths and orientations interact with different internal body shapes, as this may provide new strategies for enhancing or controlling heat transfer.
- Unconventional geometries, such as pentagonal and hexagonal bodies, have the potential to significantly influence fluid dynamics and heat distribution. Exploring these shapes may reveal novel approaches for maximizing heat transfer within enclosures.

In summary, although substantial progress has been made in understanding NCHT in square enclosures, the outlined research directions are critical for further advancing theoretical models and practical applications. Continued exploration in these areas will enable more precise control over heat transfer processes, particularly in systems influenced by internal geometries and magnetic fields, and will contribute meaningfully to both engineering and environmental technologies.

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