Review on the implementation of rotation/curvature correction function in the RANS model in predicting highly swirling flows

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Abstract: The implementation of rotation and curvature correction functions in Reynolds-Averaged Navier-Stokes (RANS) models has significantly enhanced the accuracy of turbulence predictions in complex flows, which contains strong curvature or system rotation. The conventional turbulence models, i.e. $k - \epsilon$, $k - \omega$, Spalart-Allmaras and $k - \omega$ SST, have limitations in accurately capturing the flow phenomena influenced by system rotation and streamline curvature. To overcome these deficiencies, various modifications have been proposed, including the Spalart-Shur and Smirnov-Menter corrections, which have been applied to eddy-viscosity models (EVMs). This review paper provides a comprehensive overview of the development, implementation, and performance of rotation/curvature corrections in RANS models, with a focus on their application to swirling flow such as cyclone separators and curved channels. By comparing results of the modified EVMs and conventional models against experimental and direct numerical simulation (DNS) data, this study highlights the impact of these corrections on improving the model accuracy while maintaining convenient computational cost. The results showed that modified provide a practical balance between numerical accuracy and computational cost, particularly in industrial applications that involve highly swirling flow or strong curvatures.

Keywords: turbulence model, swirling flow, rotation and curvature, eddy viscosity models.

مراجعة حول تنفيذ دالة تصحيح الدوران الانحناء في نموذج RANS في التنبؤ بالتدفقات شديدة الدوامة

الملخص: أدى تنفيذ دالة تصحيح الدوران والانحناء في نماذج رينولدز-نافير-ستوكس المتوسطة (*RANS*) إلى تعزيز دقة التنبؤات بالاضطرابات في التدفقات المعقدة، والتي تحتوي على انحناء قوي أو دوران النظام. تعاني نماذج الاضطرابات التقليدية، أي *k-ϵ* و*k-ω* و*Spalart-Allmaras* و*k-ω SST*، من قيود في التقاط ظواهر التدفق المتأثرة بدوران النظام وانحناء التيار بدقة. للتغلب على هذه العيوب، تم اقتراح تعديلات مختلفة، بما في ذلك تصحيحات *Spalart-Shur* و*Smirnov-Menter*، والتي تم تطبيقها على نماذج اللزوجة الدوامية (*EVMs*). توفر ورقة المراجعة هذه نظرة عامة شاملة على تطوير وتنفيذ وأداء تصحيحات الدوران / الانحناء في نماذج *RANS*، مع التركيز على تطبيقها على التدفق الدوامي مثل فواصل الأعاصير والقنوات المنحنية. من خلال مقارنة نتائج *EVMs* المعدلة والنماذج التقليدية مع بيانات المحاكاة العددية التجريبية والمباشرة (*DNS*)، تسلط هذه الدراسة الضوء على تأثير هذه التصحيحات على تحسين دقة النموذج مع الحفاظ على تكلفة حسابية ملائمة. أظهرت النتائج أن التعديل يوفر توازنًا عمليًا بين الدقة العددية والتكلفة الحسابية، خاصة في التطبيقات الصناعية التي تنطوي على تدفق شديد الدوامة أو انحناءات قوية.

1. Introduction

Vortex and swirling flows are widespread in a variety of mechanical systems, including dust collectors, spray dryers, vortex tubes, and combustion chambers. Furthermore, these flow phenomena are frequently observed in nature, such as tornadoes, oceanic eddies and dust devils. Consequently, the numerical simulation of such flows is of critical importance for advancing both industrial applications and scientific research to understand and analyze these flow phenomena. Swirling flows, particularly those influenced by system rotation and streamline curvature, presents a significant challenge to conventional eddy viscosity turbulence (EVMs) models to solve and close the Rynolds Averaged Navier-Stokes Equations (RANS). The full-Reynolds-stress turbulence models (RSM), detached eddy simulation (DES) and the large eddy simulation explicitly account for rotation and curvature effects in their equations. This is seen as a significant advantage compared to simpler eddy-viscosity models, which doesn't handle these effects as. Despite advances in turbulence-resolving methods like LES, DES, and RSM, the RANS models remain the most widely used in industrial computational fluid dynamics (CFD). RANS are preferred due to their balance between efficiency and accuracy, even though Reynolds Stress Models (RSMs) could theoretically be more accurate than simpler Eddy-Viscosity Models (EVMs), the robustness and the computational cost are in the favor of RANS models. Conventional Reynolds-Averaged Navier-Stokes (RANS) models, including the widely used $\boldsymbol{k-\varepsilon}$ and $\boldsymbol{k-\omega}$ models, are incapable in predicting swirling flows accurately, and hence they cannot capture the effects resulting from strong streamline curvature and rotating systems **[1, 2]**. The limitations of these models are due to the implementation of the Boussinesq hypothesis, which deal with the eddy viscosity term appear in the RANS model as an isotropic scalar **[3]**. To overcome these weaknesses, several attempts have been proposed over the years, focusing on modifying the production term in the turbulence model.

The implementation of rotation and curvature corrections in turbulence models has significantly improved the ability of RANS models to predict swirling flows. The evolution of the RANS models, from its original formulation to the modified alternatives, reflects the efforts of the RANS models to balance between the model accuracy and the computational efficiency.

This research paper aims to critically review the implementation of rotation and curvature correction functions within the Reynolds-Averaged Navier-Stokes (RANS) model. It further highlights the mathematical formulations of these modifications and compare their performance towards swirling flow phenomenon by assessing the effectiveness and accuracy of these corrections in improving turbulence modeling. It also explores their application to highly swirling flow conditions. The main objective is to identify potential enhancements and offer practical insights for better numerical flow predictions in engineering applications.

2. RANS models modifications to account for rotation/curvature effects.

2.1 Spalart and Shur modifications SARC (1997)

One of the earliest modifications was proposed by Menter in the formulation of the Shear Stress Transport (SST) model, which combined the capabilities of the k-ω model near the walls and the k-ε model in the far field **[4]**. Despite its success, the SST model still has a limitation in handling the effects of rotation and streamline curvature. In 1997, Spalart and Shur proposed a correction term to sensitize turbulence models to rotation and curvature effects **[5]**.

This correction term was applied to the Spalart-Allmaras (SA) model, the results demonstrated its efficacy in flows involving curved channels and rotating systems. The model was tested on a backward-facing step, the results demonstrated the superiority to some degree of the modified version to the conventional models when compare to experimental data.

They concluded that the proposed rotation function did not reflect a clear and strong relationship with the curvature of the streamlines. Their proposed rotation function aimed to unify rotation and curvature effects and further testing are necessary to fully understand and validate this modification in complex flows.

2.2 Hellsten modifications RCSST (1998)

Based on the Spalart-Shur correction, Hellsten **[6]** introduced modifications to the SST model, allows it to be rotationally invariant and more suitable for flows in rotating systems. This modification was particularly important for applications involving strong streamline curvature. The empirical function developed by Hellsten was calibrated for rotating channel flow with spanwise rotation, and further validated on a range of complex aerodynamic flows. They investigated their modification on a channel flow with spanwise rotation and on a boundary layer over convex-curved surface. The results showed improvement in predicting the pressure distribution and skin friction when the RCSST is used. It also showed that for the rotating channel flow the RCSST model gives accurate results for flows with rotation numbers below 0.1. For the convex-curved, the RCSST model predicts flow behavior more accurately than the conventional SST model.

2.3 Smirnov and Menter modifications SSTCC (2009)

In 2009, Smirnov and Menter implemented the Spalart-Shur correction function to the SST model, resulting in the SST with Curvature Correction (SSTCC) model **[7]**. Thess modifications improved its ability to predict swirling flows by applying the correction function to the production terms of both the turbulent kinetic energy ($\boldsymbol{k}$) and specific dissipation rate ($\boldsymbol{\omega}$) equations. The results of their work showed that the SSTCC model significantly enhance the accuracy of swirling flow predictions while maintaining the computational cost compared to more computationally expensive approaches such as Large Eddy Simulation (LES) and Reynolds Stress Models (RSMs). The model was tested on hydro-cyclone and centrifugal compressor. The results as shown in figure 3 demonstrate that the standard SST model fails in capturing the correct tangent velocity profile, which represent the near wall region “free-loss vortex” part of the Rankine vortex profile, while the modified version was in good agreement with the experimental data.

2.4 Arolla and Durbin modifications (2013)

Other researchers have also explored alternative approaches to account for rotation and curvature effects. Arolla and Durbin **[9]** proposed two approaches namely, the bifurcation approach and modified coefficients approach. The bifurcation approach adjusted to parameterize the eddy viscosity coefficient. While the modified coefficients approach parameterizes the model coefficients such that the growth rate of turbulent kinetic energy is enhanced. They validated the model on several benchmark cases and the results were encouraging in capturing the incorporate the effects of rotation and curvature.

2.5 Alahmadi and Nowakowski modifications SSTCCM (2016)

In more recent years, Alahmadi and Nowakowski. **[10]** proposed a modified version of the SSTCC model to simulate swirling flows in a cyclone separator. They introduced the Richardson number (Ri) as a simplification to avoid the use of Lagrangian derivatives, and hence a reduction in the computational cost of the model. Their findings showed that the SSTCCM model, is superior to teddy-viscosity models (EVMs) in capturing the swirling motion and vortex breakdown in cyclone flows. The modified model tested against conventional EVMs and experimental data.

The results showed that the conventional EVMs failed to capture the flow separation due to strong curvature. On the contrary, all the modified versions with the rotation function successfully capture the flow separation and reattachment.

To further examine the ability of the SSTCCM model Alahmadi and Nowakowski performed a simulation of a flow in cyclone separator. Their findings demonstrated that the only modified versions are capable of capturing the Rankine vortex profile in accordance with the experimental measurements.

In addition to that, Alahmadi et al. **[11]** performed a numerical simulation on 3D sudden expansion pipe. They examined different numerical schemes. It is found that the linear upwind scheme (LU) provides the most accurate predictions compared to experimental measurements. It has been shown that both the axial and the tangential velocity profiles predictions are in good agreements with experimental measurements. It showed that Rankine profile of the tangential velocity cannot be captured using EVMs because of the implementation of the Boussinesq hypothesis, while the SSTCCM model accurately predicts the Rankine profile of the tangential velocity, and this attributed to the use of the rotation function.

3. Mathematical Model

The fundamental physics of the fluid flow is governed mathematically by Navier-Stokes Equation, namely, the continuity equation and the mass conservation equation **[13]**. For transient incompressible flow, these equations can be expressed as follows;

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \frac{\mu}{\rho}\frac{\partial^2 u_i}{\partial x_j \partial x_j} \tag{2}$$

The Reynolds-Averaged Navier-Stokes (RANS) equations represent the time-averaged formulation of equations (1) and (2). The averaging formulation was proposed by O. Reynolds **[14]**, these equations now serve as a fundamental framework for numerous turbulence models. The RANS equations are expressed as follows:

$$\rho\left[\frac{\partial \overline{u_i}}{\partial t} + \frac{\partial \overline{u_i}\overline{u_j}}{\partial x_j}\right] = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left(2\mu S_{ij} + \tau_{ij}\right) \tag{3}$$

where S_{ij} is the time averaged strain rate tensor, and τ_{ij} is the Renolds stress tensor, and they are given by:

$$S_{ij} = \frac{1}{2}\left(\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i}\right) \tag{4}$$

$$\tau_{ij} = -\rho\overline{u_i'u_j'} \tag{5}$$

The commonly models sensitized to rotation/curvature are the Spallart-Allmaras and the Shear Stress Transport $\boldsymbol{k-\omega}$ (SST $\boldsymbol{k-\omega}$), for turbulence in swirling flows is often the Shear Stress Transport (SST) model, which combines the advantages of both the $\boldsymbol{k-\varepsilon}$ and $\boldsymbol{k-\omega}$ models. The governing equations for the SST model consist of the transport equations for the turbulent kinetic energy (k) and the specific dissipation rate (ω):

$$\frac{\partial k}{\partial t} + U_j\frac{\partial k}{\partial x_j} = P_k - \beta^* k\omega + \frac{\partial}{\partial x_j}\left[\left(v + \frac{v_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] \tag{6}$$

$$\frac{\partial \omega}{\partial t} + U_j\frac{\partial \omega}{\partial x_j} = \alpha\frac{P_k}{v_t} - \beta\omega^2 + \frac{\partial}{\partial x_j}\left[\left(v + \frac{v_t}{\sigma_\omega}\right)\frac{\partial \omega}{\partial x_j}\right] \tag{7}$$

Here, $\boldsymbol{P_k}$ represents the production term, $\boldsymbol{v_t}$ is the eddy viscosity, and $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, and $\boldsymbol{\sigma_k}$ are model constants.

To account for the effects of rotation and curvature, Spalart and Shur **[5]** proposed a correction function that modifies the production term in the k equation:

$$P_k^{modified} = P_k \cdot [1 + c_{rc} \cdot f(\Omega, S)] \tag{8}$$

where $\boldsymbol{c_{rc}}$ is a calibration constant and $\boldsymbol{f(\Omega, S)}$ is a function of the rotation rate (Ω) and strain rate ($\boldsymbol{S}$). This correction is applied to both the production term and the dissipation rate in the transport equations for (k) and (ω), which leads to accurate predictions for swirling flows **[7]**.

Hellsten **[6]** further refined this approach by introducing a modification to the specific dissipation rate equation, making the model more robust for rotating systems. The Richardson number (Ri) was introduced as a measure of the rotational effects, leading to a more efficient formulation of the correction term **[10]**. This modification has been particularly successful in applications involving cyclone separators and other systems with strong curvature or rotational motion **[11]**.

3.1 Spalart-Allmars with rotation and curvature model (SARC)

The standard one equation Spalart-Allmaras model without rotation/curvature correction can be found in **[15]**. In the standard model, the Reynolds stress tensors are related to the shear strain rate by employing the Boussinesq hypothesis as in the following formula:

$$\tau_{ij} = -\rho\overline{u_i'u_j'} = 2\mu_t S_{ij} = \rho\tilde{v}f_{v1}\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right) \tag{9}$$

The transport equation of the viscos term $(\boldsymbol{\rho\tilde{v}})$ is the given by:

$$\frac{\partial}{\partial t}(\rho\tilde{v})+\frac{\partial}{\partial x_i}(\rho\tilde{v}u_i)=C_{b1}\rho\tilde{S}\tilde{v}+\frac{1}{\sigma_v}\left[\frac{\partial}{\partial x_j}\left\{(\mu+\rho\tilde{v})\frac{\partial\tilde{v}}{\partial x_j}\right\}+C_{b2}\rho\left(\frac{\partial\tilde{v}}{\partial x_j}\right)^2\right]-Y_v \quad (10)$$

The implementation of the rotation/curvature effects in the SA model can be achieved by multiplying the production term $(\boldsymbol{C_{b1}\rho\tilde{v}})$ appear in equation (10) by the rotation function $(\boldsymbol{f_{r1}})$, which is given by:

$$f_{r1}(r^*,\tilde{r})=(1+c_{r1})\frac{2r^*}{1+r^*}[1-c_{r3}\tan^{-1}(c_{r2}\tilde{r})]-c_{r1} \quad (11)$$

The dimensionless variables $\boldsymbol{r^*}$ and $\boldsymbol{\tilde{r}}$ are given by:

$$r^*=\frac{S}{\Omega} \quad (12)$$

$$\tilde{r}=\frac{2\omega_{ij}S_{ij}}{D^4}\left(\frac{DS_{ij}}{Dt}+\left(\varepsilon_{imn}S_{jn}+\varepsilon_{jmn}S_{in}\right)\Omega_m\right) \quad (13)$$

where $\boldsymbol{S_{ij}}$, $\boldsymbol{\omega_{ik}}$, $\boldsymbol{D}$, and the empirical constants $\boldsymbol{c_{r1}}$, $\boldsymbol{c_{r2}}$, and $\boldsymbol{c_{r3}}$ are summarized in table 1.

Table 1. Strain tensors, system rotation rate and the empirical constants.

Term	Expression/value
S_{ij}	$\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$
ω_{ij}	$\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}-\frac{\partial u_j}{\partial x_i}\right)+2\varepsilon_{mji}\Omega_m$
ε_{imn}	Levi-Cvita symbol
$\frac{DS_{ij}}{Dt}$	The Lagrangian derivative
D^4	$\left(\frac{1}{2}(S^2+\Omega^2)\right)^2$
S^2	$2S_{ij}S_{ij}$

Ω^2	$2\omega_{ij}\omega_{ij}$
c_{r1}	1.0
c_{r2}	2.0
c_{r3}	1.0

3.2 Simpler version of Spalart-Allmars rotation and curvature model (SARCM)

In 2013, Zhang and Yang proposed a simpler version of the SARC model by avoiding the calculation of the Lagrangian derivative term by implementing the Richardson number Ri defined by Hellsten **[16]**. The modified model is identical to the SARC model except for the nondimensional quantity $\tilde{\boldsymbol{r}}$, which redefined as follows:

$$\tilde{\boldsymbol{r}} = \frac{\Omega}{S}\left(\frac{\Omega}{S} - 1\right) \tag{14}$$

3.3 Smirnov- Menter rotation and curvature model (SSTCC)

The SSTCC model is built on the SST $\boldsymbol{k-\omega}$ turbulence model. Smirnov and Menter implemented Spalart-Shur correction function to the production term in the transport equations of both the turbulent kinetic energy (k) and the specific dissipation rate (ω). Therefore, the production term $(\boldsymbol{P_k})$ appears in equations (6) and (7) is multiply by the rotation function $\boldsymbol{f_{rotation}}$, which is given by:

$$f_{rotation} = \max\left\{\min(f_{r1}, 1.25\ ), 0.0\right\} \tag{15}$$

where $f_{r1}$is given by equation (11). Equations (12) and (13) were used to calculate the dimensionless quantities r^* and $\tilde{r}$.

3.3 Alahmadi-Nowakowski rotation and curvature model (SSTCCM)

The SSTCCM model is a simpler version of the SSTCC model. Both models built on the SST $\boldsymbol{k-\omega}$ turbulence model. The limiter function (equation (15)) proposed by Smirnov and Menter was implemented in the SSTCCM. The dimensionless quantity $\tilde{\boldsymbol{r}}$ in equation (14) was used to avoid the calculation of the complex Lagrangian derivative term, while equation (12) used to calculate the term $\boldsymbol{r}^*$.

Table 2 listed a summary of various numerical research where the swirling flows were simulated using EVMs with rotation and curvature modifications.

Table 2. List of literatures that implements modified EVMs to numerically predicts swirling flow phenomenon in different applications.

Literature	Application	Modified EVMs	Findings	Computational cost
Chaderjian et al. [17]	NACA 0012 airfoil and a flexible UH-60A rotor	SARC	SARC model improves the boundary layer profiles for highly curved flows and helps reduce the TEV in the tip vortex cores.	5.6 hr for 210 million grid points 17.6 hr for 360 million grid points
Chaderjian [18]	V22 rotor in hover	SARC	the SARC-DES turbulence model is highly recommended over the SARC-RANS turbulence model for hover simulations	computational cost is reduced significantly by using the quick-start procedure
Shur et al. [19]	Duct with U-Turn, 3D curved channel	SARC	SARC model has demonstrated superiority over a wide range of EVMs in terms of accuracy and superior over RSM in terms of computational cost	The CPU time is 20% larger than the original SA model
Huang et al. [20]	Turbulent impinging jet heat transfer	SSTCC	In the downstream region of turbulent impinging jet, the performance of the SSTCC and original SST is similar	Not available
Ferreira et al. [21]	biradial turbine with movable guide-vanes	SARC	The SARC model is more robust and allows the application of implicit residual smoothing, which leads to faster calculation of the operating curves	Not available
Li et al. [22]	Swirling Supersonic Jets Generated Through a Nozzle-Twisted Lance	SSTCC	The numerical predictions are validated against theoretical values, and the maximum errors of the simulated Mach number and	Not available

			mass flow rate are 2.98% and 0.33%, respectively.	
Alahmadi et al. [11]	Sudden expansion pipe	SSTCCM	Conventional EVMs i.e. standard k−ϵ, RNG k−ϵ, and the SST k−ω models failed to capture the vortex breakdown phenomenon. The SSTCCM showed better performance and predicted the location of the central recirculation zone in the swirling flow	23 hours
Khaleed et al. [23]	Pressure distribution on new designed propeller	SSTCCM	The SSTCCM was implemented to numerically calculate the pressure distribution on the propeller for the upstream region.	Not available
Viken et al. [24]	Numerical simulations of airfoild and flap for DEP X 57 airplane	SARC	Wide range of RANS models were implemented to investigate the aerodynamic performance of different airfoils. For high angle attack, only the SARC and the SST models captures the recirculation region behind the upper surface of the flap.	Not available
Rogovyi et al [25]	Swirling Submerged Flow Through a Confuser	SSTCC	The SSTCC model predict the flow shape, the magnitude of the attenuation of rotation, and the velocity values in different sections in were in a good agreement with the stereo-PIV measurements.	The modified SST turbulence model predicts the main characteristics of the swirl flow using medium-power computers.
Zhang et al. [26]	Centrifugal impeller in the rotating frame of reference	SSTCC	The original SST model failed to capture the unsteady feature of the swirling flow, while the SSTCC showed better performance by predicting the flow in the centrifugal	Not available

			pump impeller, the unsteady stall and turbulence fluctuation.	
Xiaoyu et al. [27]	60 degree circularly bent channel	SSTCC & SARC	The RSM and modified models showed better performance when compared with DNS. The EVMs without modification flailed to predict the flow at the separation zone	Not available
Hreiz et al. [28]	Numerical investigation of swirling flow in cylindrical cyclones	SARC	For single tangential inlet LES and Re realizable $\boldsymbol{k}-\boldsymbol{\varepsilon}$ model that give the best results. For more than one inlet, only LES capture the shape of the velocity profile	Not available

3. Conclusion

The most popular model to numerically simulate highly swilling flows is the Reynolds Stress Model (RSM). Although RSM is computationally heavy compared to the modified RANS models, its use is more common due to its availability in many commercial software like ANSYS. All the sensitized RANS model to rotation/curvature corrections are not available in any commercial software, therefore it is not popular among researchers. The sensitized RANS models feature favorably classified as robust numerical tool for simulating complex swirling flows. It seems that these models could be further extended by saying that they could be successfully applied to other case studies such as industrial axi-centrifugal compressors or other gas turbines of comparable characteristics.

- SARC and SARCM perform well for external aerodynamic flows but still underestimate the effect of curvature at high Reynolds numbers.
- SSTCC and SSTCCM showed good performance for internal flows subjected to highly swirling flows and strong streamline curvature.
- Neither the SSTCC nor SSTCCM were examined or tested for external aerodynamic flows.

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