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A Comparative Analysis of Methodologies for Oscillation Theory in Parabolic Partial Differential Equations

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Abstract

This paper presents a comprehensive review, from 1986 to 2001, of the literature concerning the oscillatory behavior of solutions to parabolic partial differential equations with deviating arguments. We focus on the development of criteria for oscillation, highlighting the effects of discrete and continuous distributed delays, nonlinearities, forcing terms, and various boundary conditions. The review synthesizes methodologies commonly employed in the field, such as the reduction to ordinary differential inequalities and the use of integral averaging techniques. Finally, we emphasize current trends and suggest potential directions for future research.

Keywords: Oscillation; Parabolic equation; Deviating arguments.

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1. Introduction

The study of oscillatory behavior in differential equations has a rich history dating back to the pioneering work of Sturm in the 19th century on the zeros of solutions to ordinary differential equations. Classical oscillation theory for ordinary differential equations has been extensively developed over the past century, with comprehensive results established for various classes of equations including linear, nonlinear, delay, and functional differential equations. The fundamental questions addressed by oscillation theory whether all solutions exhibit persistent sign changes and under what conditions such behavior occurs have profound implications for understanding the long-term dynamics of systems modeled by differential equations.

The oscillatory behavior of solutions to differential equations has been a subject of significant attention and has motivated extensive literature over the years [2,8,12,16]. Because partial differential equations are key to modeling phenomena in science and engineering, there is a growing interest in studying their oscillatory behavior. We refer the reader to [7, 11, 13, 15, 26, 27, 29, 30, 36, 37] for parabolic equations and to [4, 14, 17–20, 23, 24, 28, 32, 34, 38] for hyperbolic equations.

In recent decades, there has been a significant extension of oscillation theory from integer to arbitrary order differential equations [1, 3, 5, 6, 9, 10, 21, 22, 25, 35], particularly for parabolic-type equations with deviating arguments. This extension represents a natural but challenging progression, as it requires addressing the additional complexities introduced by spatial variables, boundary conditions, and the interplay between temporal and spatial behaviors. The transition from ODEs to PDEs in oscillation theory has necessitated the development of new methodological approaches that can handle the infinite-dimensional nature of the problem while preserving the core philosophical framework of classical oscillation theory.

The extension to partial differential equations has been particularly fruitful for parabolic equations, where the maximum principle and spectral properties provide powerful tools for analysis. Researchers have successfully adapted techniques from ODE oscillation theory while developing novel approaches specific to the PDE context. The eigenfunction method, which reduces the spatial problem to a temporal one through integration against appropriate test functions, has emerged as a particularly effective strategy that bridges the finite and infinite-dimensional theories.

The study of oscillatory behavior in parabolic differential equations is of paramount importance in applied mathematics and theoretical analysis as oscillations are intrinsically linked to the stability of equilibrium solutions. A non-oscillatory solution converging to an equilibrium typically indicates asymptotic stability. In contrast, persistent oscillations can signify instability, the existence of limit cycles, or Hopf bifurcations, where a stable equilibrium loses stability and gives rise to a periodic orbit. Determining oscillation criteria is often a more general and powerful method than directly solving the nonlinear equation.

Understanding oscillatory behavior is a fundamental aspect of predicting the long-term dynamics, stability, and real-world manifestations of complex systems modeled by PDEs.

A solution $u(x,t)$ of a boundary value problem for an FPDE is said to be oscillatory in the domain $\Omega \times \mathbb{R}_+$ if for every positive number $T > 0$, there exists a point $(x_0, t_0) \in \Omega \times [T, \infty)$ such that $u(x_0, t_0) = 0$.

Conversely, a solution is called nonoscillatory if there exists a $T > 0$ such that $u(x,t) \neq 0$ for all $(x,t) \in \Omega \times [T, \infty)$. That is, the solution has a fixed sign (either positive or negative) for all sufficiently large time t and all points x in the spatial domain Ω.

This review focuses specifically on six seminal works that represent key milestones in the development of oscillation theory for parabolic partial differential equations with deviating arguments. The selected articles were chosen according to the following criteria:

1. Methodological Significance, as each paper significantly develops important technical approaches that have become standard in the field.

2. Chronological Progression. The selection spans the development of the field from its foundations to more recent advances, showing the evolution of ideas and techniques.

3. Comprehensive Coverage as the chosen works address the main types of equations studied in this area—nonlinear equations, forced oscillations, distributed delays, neutral equations, and mixed functional arguments.

4. Boundary Condition Variety. The collection includes results for all major boundary conditions: Dirichlet, Neumann, and Robin problems.

5. Theoretical Influence. Each paper has been highly influential, generating subsequent research and establishing directions for further development.

The papers by Yoshida (1986, 1987) form the foundation of the modern approach, establishing the eigenfunction reduction technique for equations with discrete delays. Fu and Zhang (1995) extend this framework to distributed delays using Stieltjes integrals, while Cui and Li (1998) provide the important advancement of necessary and sufficient conditions. The work by Wang and Ge (2000) addresses increasingly complex equation structures, including mixed delays. Finally, Tanaka and Yoshida (1997) represent a sophisticated treatment of multiple deviating arguments with forcing terms.

This review systematically analyzes these key contributions, examining their methodological approaches, main results, and interconnections. By understanding the development captured in these works, we can appreciate both the current state of oscillation theory for parabolic PDEs and identify promising directions for future research. The following sections provide detailed analysis of each work, comparative assessment of methodologies, and discussion of open problems that remain challenging for the field.

2. Oscillation under different Boundary Conditions

In his work [36], Yoshia establishes oscillation criteria for solutions to certain classes of nonlinear parabolic partial differential equations (PDEs) that include deviating arguments of the form:

$$(E_-) \quad u_t = a(t)\Delta u - q(x,t)f(u(x,\sigma(t))), \quad (x,t) \in \Omega \times \mathbb{R}_+,$$

$$(E_+) \quad u_t = a(t)\Delta u + q(x,t)f(u(x,\tau(t))), \quad (x,t) \in \Omega \times \mathbb{R}_+,$$

where:

- Δ is the Laplacian,
- $\Omega \subset \mathbb{R}^n$ is a bounded domain with piecewise smooth boundary,
- $a(t), q(x,t), f(s), \sigma(t), \tau(t)$ satisfy certain regularity and sign conditions (Assumptions A-I – A-VI).

by reducing the oscillation problem to the study of first-order ODE inequalities. By combining spectral theory, integral inequalities, and known ODE results, Yoshida provides verifiable criteria for oscillation or decay of solutions under various boundary conditions.

Yoshida then studied the oscillatory behavior of solutions to nonlinear parabolic partial differential equations (PDEs) with forcing terms and functional arguments in [37]. The main equation considered is:

$$u_t - a(t)\Delta u + c(x, t, u(x, t), u(x, \sigma(t))) = f(x, t), \quad (x, t) \in \Omega \times \mathbb{R}_+,$$

where: - Δ is the Laplacian, - $\Omega \subset \mathbb{R}^n$ is a bounded domain with smooth boundary, - $a(t), c, f, \sigma(t)$ satisfy certain regularity and sign conditions (Assumptions A_1–A_3).

By combining eigenfunction techniques, integration methods, and limiting conditions on the forcing term $f(x, t)$, effective criteria for forced oscillation under various boundary conditions were established.

Fu et al. [11], focused on studying the oscillatory behavior of solutions to the a nonlinear parabolic equation with a continuous distributed deviating argument and a forcing term:

$$u_t = a(t)\Delta u - \int_a^b q(x, t, \xi)F[u(x, g(t, \xi))]d\sigma(\xi) + h(x, t), \quad (x, t) \in \Omega \times (R)_+ \quad (1)$$

where:

- $\Omega \subset (R)^n$ is a bounded domain with piecewise smooth boundary $\partial\Omega$.
- $(R)_+ = [0, +\infty)$.
- $a(t) \in C((R)_+, (R)_+)$, $q(x, t, \xi) \in C(\overline{\Omega} \times (R)_+ \times [a, b], (R)_+)$.
- $F(u) \in C((R), (R))$.
- $g(t, \xi) \in C((R)_+ \times [a, b], (R))$, $g(t, \xi) \leq t$, nondecreasing in t and ξ, with $\lim_{t\to+\infty} \min_{\xi\in[a,b]} g(t, \xi) = +\infty$.
- $\sigma(\xi)$ is nondecreasing; the integral is a Stieltjes integral.
- $h(x, t) \in C(\overline{\Omega} \times (R)_+, (R))$ is the forcing term.

The analysis is conducted under three types of boundary conditions:

$$\begin{aligned}
&\text{(B1)} \quad u = \varphi, \quad (x, t) \in \partial\Omega \times (R)_+ \\
&\text{(B2)} \quad \frac{\partial u}{\partial N} = \psi, \quad (x, t) \in \partial\Omega \times (R)_+ \\
&\text{(B3)} \quad \frac{\partial u}{\partial N} + \mu u = 0, \quad (x, t) \in \partial\Omega \times (R)_+
\end{aligned}$$

where N is the unit exterior normal, and φ, ψ, μ are given continuous functions. This work generalizes the results of Yoshida on forced oscillation by considering: continuous distributed deviating arguments and non-homogeneous boundary conditions.

In 1998, Bao Tong Cui and Wei Nian Li extended the oscillation theory for partial differential equations to the oscillatory behavior for parabolic equations with multiple delays of the form

$$\frac{\partial}{\partial t}u(x,t) = a(t)\Delta u(x,t) + \sum_{k=1}^{s} a_k(t)\Delta u(x,t-\rho_k(t)) - \sum_{j=1}^{m} q_j(t)u(x,t-\sigma_j(t))$$

where $(x,t) \in \Omega \times [0,\infty) \equiv G$, Ω is a bounded domain in $\mathbb{R}^n$ with piecewise smooth boundary $\partial\Omega$, and Δ is the Laplacian operator.

The equation is considered with the Dirichlet boundary condition:

$$u(x,t) = 0, \quad (x,t) \in \partial\Omega \times [0,\infty)$$

The proof was based on the spectral properties of the Laplacian operator some results on oscillation of delay differential equations along with Green's formula and boundary conditions.

Peiguang Wang and Weigao Ge [29], Extended these previous work to include both discrete and distributed deviating arguments along with Considering three different boundary conditions:

$$u_t = a(t)\Delta u + \sum_{i=1}^{n} a_i(t)\Delta u(x,\tau_i(t)) - p(x,t)u - \int_a^b q(x,t,\xi)f(u[x,g(t,\xi)])d\sigma(\xi) + h(x,t) \tag{2}$$

where $(x,t) \in \Omega \times (R)_+$, with three types of boundary conditions:

$$\begin{aligned} &(\mathrm{B1}) \quad u = \varphi(x,t), \quad (x,t) \in \partial\Omega \times (R)_+ \\ &(\mathrm{B2}) \quad \frac{\partial u}{\partial n} = \psi(x,t), \quad (x,t) \in \partial\Omega \times (R)_+ \\ &(\mathrm{B3}) \quad \frac{\partial u}{\partial n} + \nu(x,t)u = 0, \quad (x,t) \in \partial\Omega \times (R)_+ \end{aligned}$$

The authors employed Eigenfunction methods for Dirichlet problems, Green's formulas and Jensen's inequality and Reduction to functional differential inequalities.

After that, Peiguang Wang investigated the oscillatory properties of solutions to a class of parabolic partial functional differential equations parabolic functional DEs with continuous deviating arguments and distributed deviating arguments. extending previous results that primarily focused on equations with discrete delays. The central object of study is the equation:

$$\frac{\partial}{\partial t}[u(x,t)+\lambda(t)u(x,\tau(t))]=a(t)\Delta u-c(x,t,u)-\int_a^b q(x,t,\xi)u[x,g(t,\xi)]d\sigma(\xi)+f(x,t),$$

for $(x,t)\in\Omega\times\mathbb{R}_+$, where $\Omega\subset\mathbb{R}^n$ is a bounded domain. This equation incorporates a neutral term $(\lambda(t)u(x,\tau(t)))$, a distributed delay over a continuum $[a,b]$ (modeled by a Stieltjes integral), and a nonlinearity $c(x,t,u)$.

The core methodological approach involves reducing the multi-dimensional PDE problem to a one-dimensional oscillatory problem for functional differential inequalities. Table 1 bellow summarize the different approaches of the studied papers:

Table 1: Comparison of oscillatory criteria for parabolic PDEs with deviating arguments

Reference	**Equation Type**	**Key Assumptions**	**Oscillation Criteria**
Yoshida (1986)	Nonlinear, Discrete Delay (No forcing)	$f(s)$ is convex; $\sigma(t),\tau(t)\le t$	Reduction to first-order ODE inequalities of the form $y'(t)\pm Q(t)G(t)f_2(y(g(t)))\le 0$ has no eventually positive solutions. Explicit integral conditions (e.g., $\int Q(t)dt=\infty$).

Continued on next page

Table 1 – continued from previous page

Reference	**Equation Type**	**Key Assumptions**	**Oscillation Criteria**
Yoshida (1987)	Nonlinear, Forced, Discrete Delay	Standard regularity on coefficients and delays.	$\liminf \int \tilde{H}(t)dt = -\infty$, $\limsup \int \tilde{H}(t)dt = +\infty$, where $\tilde{H}(t)$ incorporates the forcing $f(x,t)$ and boundary data (ϕ or ψ).
Fu & Zhang (1995)	Nonlinear, Forced, Distributed Delay	F convex, odd; $g(t,\xi) \leq t$, nondecreasing; $\sigma(\xi)$ nondecreasing.	Differential inequalities (I1, I2) have no eventually positive solution. For (B1): $\liminf \int H(t)dt = -\infty$, $\limsup \int H(t)dt = +\infty$ ($H(t)$ includes forcing and boundary data).
Cui & Li (1998)	Linear, Multiple Discrete Delays (No forcing)	Dirichlet boundary condition ($u = 0$).	A necessary and sufficient condition: The associated delay differential inequality $V'(t) + \alpha_0 a(t)V(t) + ... \leq 0$ has no eventually positive solution.

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Table 1 – continued from previous page

Reference	Equation Type	Key Assumptions	Oscillation Criteria
Wang & Ge (2000)	Nonlinear, Forced, Mixed Delays (Discrete + Distributed)	f convex, odd; $\tau_i(t) \leq t$, $g(t,\xi) \leq t$, both nondecreasing.	Associated differential inequality has no eventually positive solution. For (B1): $\liminf \int H(s)ds = -\infty$, $\limsup \int H(s)ds = +\infty$ ($H(s)$ includes forcing and boundary data).

3. Comparative analysis and methodologies

The reviewed literature demonstrates a clear evolution in the study of oscillation criteria for parabolic partial differential equations with deviating arguments. The development can be analyzed through several dimensions:

3.1. Chronological development of techniques

- **Yoshida (1986)** established the foundational approach by reducing PDE problems to ordinary differential inequalities, focusing on discrete delays without forcing terms.
- **Yoshida (1987)** extended this framework to include forcing terms, introducing limit conditions on integrals of the transformed forcing function.
- **Fu & Zhang (1995)** generalized the theory to distributed delays using Stieltjes integrals, significantly expanding the class of admissible functional arguments.
- **Cui & Li (1998)** provided the first necessary and sufficient conditions for oscillation, specifically for linear equations with multiple discrete delays.

- **Wang & Ge (2000)** combined discrete and distributed delays in a unified framework, while also incorporating more complex boundary conditions.
- **Wang (2001)** addressed neutral-type equations, representing the most complex functional form among the reviewed works.

Methodology

The progression shows a movement from:

- Simple to complex functional arguments (discrete → distributed → mixed → neutral)
- Homogeneous to non-homogeneous equations (unforced → forced)
- Sufficient conditions to necessary and sufficient conditions
- Simple to complex boundary conditions (Dirichlet → Neumann → Robin)

Methodological approaches

All studies employ a similar reductionist approach with the following core components: Eigenfunction Reduction Technique that involves:

$$U(t) = \frac{\int_\Omega u(x,t)\Phi(x)dx}{\int_\Omega \Phi(x)dx}$$

where $\Phi(x)$ is the first eigenfunction of the Laplacian operator for the corresponding boundary value problem. This transformation reduces the spatial PDE to a temporal functional differential inequality.

Green's Formula Application to handle the Laplacian term:

$$\int_\Omega \Delta u \cdot \Phi dx = \int_\Omega u \cdot \Delta\Phi dx + \text{boundary terms}$$

This step is crucial for incorporating boundary conditions into the resulting ordinary differential inequality.

Jensen's Inequality For nonlinear problems employed to handle convex nonlinearities:

$$f\left(\frac{1}{|\Omega|}\int_\Omega udx\right) \leq \frac{1}{|\Omega|}\int_\Omega f(u)dx$$

This allows the treatment of nonlinear terms in the reduced inequality.

Variations in Methodological Approach:

Table 2: Methodological variations across studies

Study	**Equation Type**	**Methodological Innovations**
Yoshida (1986)	Nonlinear, discrete delay	Established the basic eigenfunction reduction framework for delay parabolic equations
Yoshida (1987)	Forced, discrete delay	Incorporated forcing terms through limit conditions on $\int G(t)dt$
Fu & Zhang (1995)	Distributed delay	Extended methodology to Stieltjes integrals for continuous delay distributions
Cui & Li (1998)	Multiple delays	Developed techniques for necessary and sufficient conditions
Wang & Ge (2000)	Mixed delays	Combined discrete and distributed delay treatment in unified framework

4. Open problems and future research directions

From this analysis, several promising research directions and open problems emerge. As Current theories predominantly assume convex nonlinearities applying Jensen's inequality, the extension to more general nonlinearities remains largely open. Also, finding alternatives to Jensen's inequality that preserve the reduction from PDE to ODE inequality. On the other hand few research focuses exclusively on higher order parabolic equations, so extension and further investigations remain open. The extension of the theory to almost periodic coefficients or Random coefficients is a good research direction

5. Conclusion

The field of oscillation theory for parabolic PDEs with deviating arguments, while well-developed for certain classes of problems, presents numerous open challenges and research opportunities. The most promising directions appear to be: extending beyond convex nonlinearities, handling more complex functional structures, addressing systems and higher-order equations, and developing computational approaches and physical applications.

Future research in these directions would not only advance the theoretical foundations but also enhance the applicability of oscillation theory to concrete problems in science and engineering where delayed feedback and spatial diffusion interact to produce complex dynamics.

References

[1] Abbas. S., Benchohra. M. and Graef. J. R. (2021). Osillation and nonoscillation results for the Caputo Fractional q-Differential Equations and Inclusions, *Journal of Mathematical Sciences,* Vol. 258, No. 5, pp. 577-593.

[2] Agarwal, R.P., Bohner, M., Grace, S.R. and O'Regan, D. (2005). Discrete Oscillation Theory, *Hindawi Publishing Corporation: New York, NY, USA.*

[3] Boudjerida, A., Seba, D., Meskine, N. et al. Oscillation of non-instantaneous impulsive hybrid-fractional delay differential inclusions with Hilfer-Katugampola derivative. J. Appl. Math. Comput. (2025). https://doi.org/10.1007/s12190-025-02541-w

[4] Cui, B., Lalli, B.S. & Yu, Y. Forced oscillations of hyperbolic differential equations with deviating arguments. Acta Mathematicae Applicatae Sinica 11, 369–377 (1995). https://doi.org/10.1007/BF02007175

[5] Chatzarakis GE, Nagajothi N, Deepa M, Sadhasivam V. On the Oscillatory Behavior of a Class of Mixed Fractional-Order Nonlinear Differential Equations. Symmetry. 2025; 17(3):446. https://doi.org/10.3390/sym17030446

[6] Chatzarakis, G.E., Logaarasi, K. Forced oscillation of impulsive fractional partial differential equations. Partial Differential Equations in Applied Mathematics Volume 7, June 2023, 100478. https://doi.org/10.1016/j.padiff.2022.100478

[7] L. D. Du Oscillation of a quasilinear impulsive delay parabolic equation with two different boundary conditions, J. Math. Anal. Appl. 307 (2005) 524–532. doi:10.1016/j.jmaa.2004.09.057

[8] L.H. Erbe, Q. Kong, B.G. Zhang, Oscillation Theory for Functional Differential Equations, Marcel Dekker, New York, 1995.

[9] Feng Q., Meng F., Oscillation of solutions to nonlinear forced fractional differential equations, Electr. J. Diff. Equ., 2013, 169: 1-10.

[10] Feng, Q. and Liu, A. (2019) Oscillation for a Class of Fractional Differential Equation. Journal of Applied Mathematics and Physics, 7, 1429-1439. doi: 10.4236/jamp.2019.77096.

[11] X.L. Fu and L.Q. Zhang, Forced oscillation for certain nonlinear delay parabolic equations, J. Partial Diffi Eqs. 8, 82-88, (1995).

[12] A. Halanay, Volume 23: Differential Equations: Stability, Oscillations, Time Lags, Mathematics in Science and Engineering Book series, Pages ii-x, 1-528 (1966)

[13] J. Jaroš T. Kusano, N. Yoshida Oscillation properties of solutions of a class of nonlinear parabolic equations, Journal of Computational and Applied Mathematics 146 (2002) 277–284

[14] K. Kreith, T. Kusano, N. Yoshida Oscillation properties of nonlinear hyperbolic equations SIAM J. Math. Anal., 15 (1984), pp. 570-578

[15] Kong, H. and Xu, R. Forced oscillation of fractional partial differential equations with damping term, Fractional Differential Calculus Volume 7, Number 2 (2017), 325-338 doi:10.7153/fdc-2017-07-15

[16] Kusano T. and Naito M., Oscillation criteria for a class of perturbed schrôdinger equations, Canad. Math. Bull. Vol. 25 (1) (1982).

[17] B.S. Lalli, Y.H. Yu, B.T. Cui Oscillations of certain partial differential equations with deviating arguments Bull. Austral. Math. Soc., 46 (1992), pp. 373-380

[18] B.S. Lalli, Y.H. Yu, B.T. Cui Oscillations of hyperbolic differential equations with functional arguments Applied Mathematics and Computation 53:97-110 (1993). https://doi.org/10.1016/0096-3003(93)90095-V

[19] B.S. Lalli, Y.H. Yu, B.T. Cui, Forced oscillations of hyperbolic differential equations with deviating arguments, Indian J. Pure Appl. Math. 25 (1994) 387-397.

[20] B.S. Lalli, Y.H. Yu, B.T. Cui Forced oscillations of the hyperbolic differential equations with deviating arguments Indian J. Pure Appl. Math., 25 (4) (1995), pp. 387-397

[21] Li, Wei Nian, Sheng, Weihong. "Oscillation properties for solutions of a kind of partial fractional differential equations with damping term." Journal of Nonlinear Sciences and Applications, 9, no. 4 (2016): 1600–1608

[22] Ma Q-X., Liu K-Y., Liu A-P., Forced oscillation of fractional partial differential equations with damping term, J. of Math. (PRC) Vol. 39 (2019) No. 1

[23] D.P. Mishev Oscillatory properties of the solutions of hyperbolic differential equations with “maximum” Hiroshima Math. J., 16 (1986), pp. 77-83

[24] Pagan G. An oscillation theorem for characteristic initial value problems in linear hyperbolic equations, Proceedings of the Royal Society of Edinburgh, 77 A, 265-271, 1977

[25] A. Raheem, Md. Maqbul, Oscillation criteria for impulsive partial fractional differential equations, Comput. Math. Appl. 73(8) (2017) 1781-1788, https://doi.org/10.1016/j.camwa.2017.02.016.

[26] Y. Shoukaku Oscillation of the solutions of parabolic equations with nonlinear neutral terms J. Math. Anal. Appl. 326 (2007) 556–569. doi:10.1016/j.jmaa.2005.09.048

[27] Tanaka, S. Yoshida, N. Oscillations of solutions to parabolic equations with deviating arguments, Tamkang Journal of Mathematics Volume 28, Number 3, Autumn 1997.

[28] P.G. Wang Forced oscillation of a class of delay hyperbolic equations boundary value problem Appl. Math. Comput., 103 (1) (1999), pp. 15-25

[29] P.G. Wang, W.G. Ge Oscillations of a class of functional parabolic differential equations Appl. Math. Lett., 13 (7) (2000), pp. 85-91

[30] Wang, P. G., On the oscillation of solutions of parabolic partial functional differential equations. Mathematica Slovaca 51.4 (2001): 449-458. http://eudml.org/doc/32168.

[31] P.G. Wang, W.G. Ge Oscillation of a class of hyperbolic equations Appl. Math. Comput., 116 (1) (2000), pp. 101-110

[32] P.G. Wang, Y.H. Yu Oscillation of a class of hyperbolic boundary value problem Appl. Math. Lett., 10 (7) (1999), pp. 91-98

[33] P. Wang, Y. Wu, L. Caccetta, Oscillation criteria for boundary value problems of high-order partial functional differential equations J. Comput. Appl. Math., 206 (1) (2007), pp. 567-577

[34] P.G. Wang, J.L. Zhao, W.G. Ge Oscillation criteria of nonlinear hyperbolic equations with functional argument Comput. Math. Appl., 40 (5) (2000), pp. 513-521.

[35] Xu, D., Meng, F. Oscillation criteria of certain fractional partial differential equations. Adv Differ Equ 2019, 460 (2019). https://doi.org/10.1186/s13662-019-2391-y

[36] Yoshida, N., Oscillation of nonlinear parabolic equation with functional arguments, Hiroshima Math. J., 16(1986), 305–314.

[37] Yoshida, N., Forced oscillations of solutions of parabolic equations, Bull. Austral. Math. Soc., 36 (1987),289–294.

[38] Norio Yoshida, An oscillation theorem for characteristic initial value problems for nonlinear hyperbolic equations, Proc. Amer. Math. Soc., 76 (1979), 95–100